Murat Buda*

Prime Numbers and Geometry

February 7, 2016

*muratbuda@hotmail.com
www.muratbuda.com

PINARBAŞ PRINTERY

Prime number is noble number.

Murat Buda

Contents

1 INTRODUCTION 11

2 PRIME and PENTAGONAL NUMBERS 13

2.1 Some Properties of $\mathbf{U_n}$(Pentagonal Numbers) 28

2.2 Studying The Quantities of $\mathbf{Q_k}$ Numbers $[\lambda(\mathbf{Q_k^2})]$ 30

3 GEOMETRIC ANALYSIS 37

3.1 Choosing the $(\mathbf{m})$ Value 53

3.2 On Ellipse, $\lambda(\mathbf{p})$ is Perpendicular to Major Axis 58

3.3 $\lambda(p)$ and $\lambda(q)$ are Perpendicular: $[\lambda(\mathbf{p}) \perp \lambda(\mathbf{q})]$ 61

3.4 Limit Values 65

3.5 Finding a Composite Number out of the Prime Number 71

3.6 Finding Sum $\mathbf{Q_n}$, According to The Number $\mathbf{10^n}$ 74

4 NUMERICAL MODELING 75

4.1 Find the Locus Curve $[y = G(x)]$ 75

4.2 Studying the Optimum **m** Values 81

4.3 Two Domains Analysis with a Piecewise Function 86

4.3.1 Results-A 88

4.3.2 Results-B 90

5 Conclusion 93

6 Appendix 95

Foreword

After approximately 2 years of ardous work, this book came to light to publish. Hence I owe immense appreciation to Hasan Koç and Gülsüm Deniz for their precious help in preparation of the data lists. It gives me great pleasure in acknowledging the support and help of Semih Cengiz[1] for his brilliant graphic designs. In addition I am also indebted to Mustafa Turkoz[2] for contributing to the book via MATLAB programming as well as being the editor of this book. Also, I'm very much obliged to E. Ezgi Atar[3] for impressive translation to English of this book. Finally, I would like to express my deepest gratitude to Professor Taylan Akdoğan[4] for Two Domains Analysis with a Piecewise Function.

[1] semihcengizx@hotmail.com
[2] mustafa.turkoz@boun.edu.tr
[3] ezgi.atar@boun.edu.tr
[4] taylan.akdogan@boun.edu.tr

Abstract

Counting of prime numbers is one of the oldest and interesting problems in mathematics. Throughout the history, many mathematicians have studied on number theory and prime numbers and they have provided many significant solutions. In this study, we aim to present an alternative model. In this model, we handle the distribution of prime numbers using a geometric method.

Chapter 1

INTRODUCTION

I was studying at Vefa High School between the years 1963-1966. I was mostly interest in Algebra and Geometry classes. Therefore, in the last year of the high school, my deceased Math teacher, Mr. Ihsan Irk advised me to engage in number theory if I could find time in the future. I could not spare enough time to this subject back then, as I was preparing for the entance exams of Istanbul Technical University. A few years after I was accepted to the university, I started to study number theory. I gave all the algebra and geometry problems that I was interested, except one. I mostly paid attention to a locus problem and number theory.

I have published some problems on number theory, geometry and algebra on my web site (muratbuda.com). There were not many books and articles on mathematics back then, and I examined all of the books I could find about number theory. After I examined the written sources about prime numbers, I focused my attention on them. I also published my researches on my web site.,

On one of the books about irregularity of prime numbers that I read, I learnt that there was no formula that can generate all n‘ s in a single formula. As I was interested in relatively prime numbers, I encountered an interesting issue about them. I believe that the people who are interested in this study I publish as three chapters, are going to take this subject much further.

Chapter 2

PRIME and PENTAGONAL NUMBERS

In one of my studies on relatively prime numbers, I saw that if we subtract 1 from the square of a prime number, it can be divided by 24. The divisibility by 24 starts with 5.

Respectively:

5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41.... and it goes on . The emerging numbers after the division by 24 are:

1, 2, 5, 7, 12, 15, 22, 35, 40, 57, 70.....

I saw on the mathematics books that the similar series above are used in the sum of the divisors of the numbers. However, I learnt from the issue 2014-II of Matematik Dünyası that these are pentagonal numbers

$$S = (1-x)(1-x^2)(1-x^3)(1-x^4).....$$

At the same time, multiplication of finitely many terms and infinitely many multiplicative:

$$S = 1-x-x^2+x^5+x^7-x^{12}-x^{15}+x^{22}+x^{26}-x^{35}....$$

The series stated above is found. By the exponentiation of the series, we write down the differences:

(A) :1, 2, 5, 7, 12, 15, 22, 26, 35, 40, 51, 57, 70, 77, 92, 100...

(B) : 1 **3** 2 **5** 3 **7** 4 **9** 5 **11** 6 **13** 7 **15** 8...

As you can see, there are two interpenetrating number sequences. Natural numbers are followed by odd numbers in an order. Something attracted my attention here. There were more numbers in the last series with 1.2.5.7... We may show what is written by a formula. The series is U_n, and the numbers where U_n is placed are Q_n:

$Q_n^2 - 1 = 24U_n$

$$Q_n = \sqrt{24U_n + 1} \tag{2.1}$$

If we replace all U_n values found in the series in (2.1)

As Q_n We find the sequences

(C): 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, 41, 43, 47, 49, 53...

In this sequence, apart from the prime numbers, we see these numbers:

25, 35, 49, 55, 65, 77, 85, 91, 95, 115, 119, 121, 125 ... It is clear that these numbers are product of prime numbers. There are no even numbers, 3 and the multiples of 3 in the sequence (C). In short, this is a sequence without the multiples of 2 and 3. In this sequence, we can indicate that the squared numbers minus one can be divided by 24.

Since Q_n is an odd number :

$Q_n = 2k + 1$

$Q_n^2 = 4k^2 + 4k + 1$

$Q_n^2 - 1 = 4k(k+1)$

Either k or $k+1$ has to be even. Then $Q_n^2 - 1$, is divided by 8

$Qn^2 - 1 = (Q_n - 1)(Q_n + 1)$

Since one of the consecutive numbers like $(Q_n - 1)$, Q_n, (Q_{n+1}) is divided by 3 and , Q_n cannot be divided by 3, either $Q_n - 1$ or the necessary cancellations are done $Q_n + 1$ is divided by 3. As a result, $Q_n^2 - 1$ is divided by 24, the product of the multiplication of 8 by 3. Now we have two series.

1-The sequence U_n

2–The sequence Q_n: This provides not only prime numbers but also the products of prime numbers multiplied.

Table 1

U_n	Q_n	U_n	Q_n	U_n	Q_n
1	5	51	35	176	65
2	7	57	37	187	67
5	11	70	41	210	71
7	13	77	43	222	73
12	17	92	47	247	77
15	19	100	49	260	79
22	23	117	53	287	83
26	25	126	55	301	85
35	29	145	59	330	89
40	31	155	61	345	91

Just out of curiosity, I wanted to examine some U_n sequences without prime numbers.

Table 2

U_n	Q_n	U_n	Q_n
26	25	590	119
51	35	610	121
100	49	651	125
126	55	737	133
176	65	852	143
247	77	876	145
301	85	1001	155
345	91	1080	161
376	95	1190	169
551	115	1276	175

In this table, I paid attention to the numbers that have 1 and 6 as the last figures. We write these numbers again:

(D) : 26, 51, 126, 176, 301, 376, 551, 651, 876, 1001...

If we subtract 1 from these numbers, the remainder can be divided by 25. .

As you can see, we also have the sequence U_n here. 25 is the square of 5, but 24 plus 1 also equals 25.

26-1=1.25	376-1=15.25
51-1=2.25	551-1=22.25
126-1=5.25	651-1=26.25
176-1=7.25	876-1=35.25
301-1=12.25	1001-1=40.25

25=24+1

Now we replace 25 with 24+1 on the table above.

26-1 = 24+1	51-1 = 2.(24+1)
26 = 24+1+1	51 = 2.24+2+1

We also write the other ones this way.

$$26 = 1.(24+1)+1, = 1.24+1+1, = 1.1.24+1+1$$

$$51 = 2.(24+1)+1 = 2.24+1+2 = 1.2.24+1+2$$

$$126 = 5.(24+1)+1, = 5.24+1+5, = 1.5.24+1+5$$

$$176 = 7.(24+1)+1, = 7.24+1+7, = 1.7.24+1+7$$

$$301 = 12.(24+1)+1 = 12.24+1+12 = 1.12.24+1+12$$

As we can see in the last equation, the elements of the sequence U_n with no prime numbers consist of the product of previous U_n 's plus the same U_n 's.

If we apply this system to the other U_n 's s with no prime numbers:

$$100 = 2.2.24+2+2$$

$$247 = 2.5.24.+2+5$$

$$345 = 2.7.24+2+7$$

$$590 = 2.12.24+2+12$$

$610 = 5.5.24 + 5 + 5$

$737 = 2.15.24 + 2 + 15$

$852 = 5.7.24 + 5 + 7$

The U_n 's with prime numbers are U_p, the U_n's without prime numbers are U_q. U_m and U_r are the terms before U_q

$$U_q = 24U_m.U_r + U_m + U_r \tag{2.2}$$

U_m and U_r can be any two terms before U_q, $U_m = U_r$ is possible.

For larger U_q 's if we assume $U_q > U_r, U_s, U_t$:

$$U_q = 24^2 U_r.U_s.U_t + 24(U_r.U_s + U_r.U_t + U_t.U_s) + U_r + U_s + U_t \tag{2.3}$$

In addition to the existence of U_q' s in such a structure, this type of U_q' s can lead us to the infinity.

The Formula (2.1)

$Q_n = \sqrt{24U_n + 1} \qquad U_n = U_q$

$Q_n = \sqrt{24U_q + 1}$

$U_q = 24U_mU_r + U_m + U_r$

$Q_n = \sqrt{24(24U_mU_r + 24U_m + U_r) + 1}$

$Q_n = \sqrt{24^2U_mU_r + 24U_m + 24U_r + 1}Q_n \qquad = \sqrt{(24U_m + 1)(24U_r + 1)}$

$Q_m = \sqrt{24U_m + 1}$

$Q_r = \sqrt{24U_r + 1}$

$Q_n = Q_m.Q_r$

As Q_n is equivalent to the multiplication of the two terms coming before itself, the number , which will be given by the formula (2.1) of this U_q is not prime either. Applying the same system, we also prove that $Q_n = Q_m.Q_s.Q_t...$ In the sequence Q_n there are both prime numbers and the composite numbers, which are product of prime numbers, in particular, prime numbers. Likewise, in the sequence U_n , we both have U_p 's with prime numbers and U_q 's without prime numbers. Then, in this sequence, apart from the U_p 's with prime numbers, U_q 's are also composite U_n 's. In short, there are prime U_p 's and composite U_q 's.

To sum up:

1 We have the sequences U_n

2 We learn the reason why some terms here do not provide prime numbers in the formula (2.1). Now we take a break in this matter and examine the sequence Un, instead

$S = 1 - x - x^2 + x^5 + x^7 - x^{12} - x^{15}...$

$S = 1.x^0 - x^1 - x^2 + x^5 + x^7 - x^{12} - x^{15}...$

For the exponents of (x)

0	1	2	5	7	12	15	22	26	35	40	51	57	70

1	1	3	2	5	3	7	4	9	5	11	6	13

$0+1=1$	$=U_1$
$1+1=2$	$=U_2$
$1+1+3=5$	$=U_3$
$1+1+3+2=7$	$=U_4$
$1+1+3+2+5=12$	$=U_5$
$\ldots\ldots\ldots\ldots\ldots$	
$1+1+3+2+5+3+7+4+9+.....+2n-1+n$	$=U_n$

$$1+2+3+4+5+6+............+n = \frac{n(n+1)}{2}$$

$$1+2+3+4+5+6+..........n-1 = \frac{n(n-1)}{2}$$

$$1+3+5+7+................+2n-1 = n^2$$

$$U_n = \frac{3n^2 \pm n}{2} \tag{2.4}$$

The pair of U_n for each n. For example

$$n_1 = 1 \;\; U_1 = \frac{(3-1)}{2} = 1 \qquad\qquad U_2 = \frac{(3+1)}{2} = 2$$
$$n_1 = 2 \;\; U_3 = \frac{(3.4-2)}{2} = 5 \qquad\qquad U_4 = \frac{(3.4+2)}{2} = 7$$

The sequence Q_n is a sequence without the multiples of 2 and 3. We can write a sequence with the multiples of 2 and 3 as follows;

$$R_n = 2.3.n = 6n$$

If we add ± 1, ± 5, ± 7, ... to term $R_n = 6n$ we find the sequences without the multiples of 2 and 3. If we want the first term of this sequence to start from 5, the sequence $Q_n = 6n \pm 1$ will be useful. Then:

In the formula $Q_n = \sqrt{(24U_n + 1)}$ if we replace $U_n = \dfrac{3^2 \pm n}{2}$,

$$Q_n = \sqrt{24\frac{3n^2 \pm n}{2} + 1}$$
$$Q_n = \sqrt{36^2 n \pm +12n + 1}$$
$$Q_n = \sqrt{(6n \pm 1)^2}$$
$$Q_n = 6n \pm 1 \tag{2.5}$$

We create a table by numerating the terms of the sequence U_n

$U_1 = 1$, $U_2 = 2$, $U_3 = 5$... and $Q_1 = 5$, $Q_2 = 7$, $Q_3 = 11$... Since there are two Since there are two U_n for each n :

For $\quad n = 1 \quad k_1 = 1 \quad k_2 = 2 \quad n = \dfrac{k}{2} \qquad\qquad k$: Even

For $\quad n = 2 \quad k_3 = 3 \quad k_4 = 4 \quad n = \dfrac{k+1}{2} \qquad\qquad k$: Odd

If we numerate as in the example, hereafter we use U_k instead of U_n and Q_k instead of Q_n Here, k gives the sequence number k .

No: n	**Sequence: k**	**U_n**
1	1	1
	2	2
2	3	5
	4	7
3	5	12
	6	15

For $Q_1 = 5$, we indicate U_k (which are (U_q)'s without prime numbers :

$5 : U_8, U_{11}, U_{18}, U_{21}, U_{28}....$

For $Q_2 = 7$:

$7 : U_{11}, U_{16}, U_{25}, U_{30}, U_{39}, U_{44}....$

Continuing these examples and writing the value k only, we create a table:

Table 3

5	8	11	18	21	28	31	38	41	48	51	58	61	68	71	78	81
7	11	16	25	30	39	44	53	58	67	72	81	86	95	100	109	114
11	18	25	40	47	62	69	84	91	106	113	129	135	150	157	172	179
13	21	30	47	56	73	82	99	108	125	134	151	160	177	186	203	212
17	28	39	62	73	96	107	130	141	164	175	198	209	232	243	266	277
19	31	44	69	82	107	120	145	158	183	196	221	234	259	272	297	310
23	38	53	84	99	141	145	176	191	222	235	268	283	314	329	360	375
25	41	58	91	108	164	158	191	208	241	258	291	308	341	358	391	408
29	48	67	106	125	175	183	222	241	280	299	338	357	396	415	454	473
31	51	72	113	134	198	196	237	278	299	340	361	402	423	464	485	526
35	58	81	128	151	209	221	268	291	338	361	408	431	478	501	548	578
37	61	86	135	160	232	234	283	308	357	382	431	456	505	530	579	604
41	68	95	150	177	243	259	314	341	396	423	478	505	560	587	642	669
43	71	100	157	186	266	272	329	358	415	444	501	530	587	616	673	702
47	78	109	172	203	277	297	360	391	454	485	548	579	642	673	736	767
49	81	114	179	212	400	310	375	408	473	506	571	604	669	702	767	800

If we look at shaded the diagonal line, it indicates the products of 8. If we divide these diagonal numbers by 8, we find the sequence 1, 2, 5, 7, 12, 15, 22...

Considering the differences among row corresponding to $Q_1 = 5$,

it continues as 3 and 7.

The rows corresponding to $Q_2 = 7$ are 5 and 9.

The rows corresponding to $Q_3 = 11$ are 7 and 15 It goes on this way. Considering these differences, we rearrange the table:

Table 4

k	Q_k	U_k	a	b	c	d	e	f	g	h	i	y	k	l	m
1	5	1	3	7	8	11	18	21	28	31	38	41	48	51	58
2	7	2	5	9	11	16	25	30	39	44	53	58	67	72	81
3	11	5	7	15	18	25	40	47	62	69	84	91	106	113	128
4	13	7	9	17	21	30	39	56	73	82	99	108	117	134	143
5	17	12	11	23	28	39	62	73	96	107	130	141	164	175	198
6	19	15	13	25	21	34	57	82	95	120	133	158	171	196	209
7	23	22	15	31	28	43	84	99	130	145	176	191	222	237	268
8	25	26	17	33	31	48	75	108	125	158	175	208	22	258	275
9	29	35	19	39	38	57	106	125	164	183	222	241	280	299	338
10	31	40	21	41	41	62	93	134	155	196	217	28	279	320	341
11	35	51	23	17	48	71	98	121	138	161	178	201	218	241	258
12	37	57	25	49	51	76	111	160	185	234	259	308	333	382	407
13	41	70	27	55	58	95	150	177	232	259	314	341	396	423	478

A) Columns

1. In the column (a), a sequence of odd numbers starting from 3.
2. In the column (b) a sequence starting from 7 and in succession with the differences 2 and 6.
3. In the column (c) a sequence starting from 8 and in succession with the differences 3 and 7.

B) Rows

1. The number 3 in the column (a) plus the number 8 in the column (c) equals the number 11 in the column (d). Then, the number 7 in the column (b) plus the number 18 in the column (e) equals the number 18. Again, plus 3 equals the number 21 in the column (f). Continuing this way, we find the other numbers
2. Corresponding to $Q_k = 7$ 1. the number 5 in the column (a) plus the number 11 in the column (c) equals 16. Then, 16 plus 9 in the column (b) equals the number 30 in the column (e). We proceed with ordering this way.

All these values (k) of the numbers U_k we find are the terms of the sequence without prime numbers. The numbers placed in the shaded diagonal are the products of that U_n times 8. The numbers placed on the left and right sides of this diagonal are symmetrical. For example:

The number $U_{11} = 51$ gives us $Q_{11} = 35$. As , we see the sequence number $35 = 5.7$, we see the sequence number $k = 11$ in the column (d) by $Q_1 = 5$ and in the column (c) by $Q_2 = 7$ If we pay attention to the diagonal 8-16, kboth the numbers 11 are symmetrical. Then, if we add the number in the column (b) to the diagonal lines, we do not need to consider the numbers before the diagonal. For example:

The numbers placed on the diagonal in the **Table 4** deare placed in the column (c) in the **Table 5** . In the row, $Q_1 = 5$ sin the column (a) plus 8 in the column (c) equals 11 and 11 plus 7 equals 18. Then, it continues again by addition of 3 and 7 respectively. In the row $Q_2 = 7$ s, 9 in the column (b) plus 16 in the column (c) equals 25 and 25 plus 5 in the column (a) equals 30. Then 30 plus 9 again equals 39. Placed in the row, $Q_3 = 11$, the number 7 in the column (a) plus 40 in the column (c) equals 47, and 47 plus 15 in the column (b) equals 62. The additions by the numbers in the column (c) and the numbers in the column (a) and (b) continue diagonally. (The first numbers to add are shaded)

Table 5

k	Q_k	U_k	a	b	c	d	e	f	g	i	y	k	l
1	5	1	3	7	8	11	18	21	28	31	38	41	58
2	7	2	5	9	16	25	30	39	44	53	58	67	72
3	11	5	7	15	40	47	62	69	84	91	106	113	128
4	13	7	9	17	56	73	82	99	108	125	134	151	160
5	17	12	11	23	96	107	130	141	164	175	198	209	232
6	19	15	13	25	120	145	158	183	196	221	234	259	272
7	23	22	15	31	176	191	222	237	268	283	314	329	360
8	25	26	17	33	208	241	258	291	308	341	358	391	424
9	29	35	19	39	280	299	338	357	396	415	454	473	512
10	31	40	21	41	320	361	382	423	444	485	506	547	568
11	35	51	23	47	408	433	478	501	548	571	618	641	688
12	37	57	25	49	456	505	530	579	604	653	678	727	752
13	41	70	27	55	560	587	642	669	724	751	806	833	888

The values in the **Table 4** are the k values of U_k değerleridir. We can also find the U_k values directly. According to (2.2)

$U_q = 24U_m.U_r + U_m + U_r$

$U_q = (24U_r + 1).U_m + U_r$

$Q_r^2 = 24U_r + 1$

$$U_q = Q_r^2.U_m + U_r \tag{2.6}$$

If we write this in the general sense, we find this formula:

$$U_{q_k}|_{k=r}^{\infty} = Q_r^2.\, U_k|_{k=r}^{\infty} + U_r \tag{2.6a}$$

This sequence gives us the U_q without prime numbers. According to (2.6), we create Table 6.

Table 6

U_k	U_{q_k}	1	2	5	7	12	15	22
1	$25U_{q_k}+1$	26	51	126	176	301	376	551
2	$49U_{q_k}+2$	51	100	247	345	590	737	1080
5	$121U_{q_k}+5$	126	247	610	852	1457	1820	2667
7	$169U_{q_k}+7$	17	6345	852	119	2035	2542	3725
12	$289U_{q_k}+12$	301	590	1657	2035	3480	4357	6370
15	$361U_{q_k}+15$	376	737	1820	2542	4377	5430	7957
22	$529U_{q_k}+22$	551	1080	1667	3725	6370	7957	11660
26	$625U_{q_k}+26$	651	1276	3151	4401	7526	9401	13776
35	$841U_{q_k}+35$	876	1717	4240	5922	10127	12650	18537
40	$961U_{q_k}+40$	1001	196	2 4845	6767	11572	14455	31182

For example, if we start the expression $121U_k + 5$ in the third row from $U_1 = 1$ and $U_2 = 2$, we find U_q which is placed before the first and second rows (such as 126 and 247) here. Instead, if we start each term U_q from the U_q next to it, we do not see the previous U_q.

We create **Table 7** this way

Table 7

k	U_k	Dizin	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	$25U_k+1$	26	51	126	176	301	376	551	651	876	1001	1276	1426	1751
2	2	$49U_k+2$	100	247	345	590	737	1080	1276	1717	1962	2501	2795	3432	3775
3	5	$121U_k+5$	610	852	1457	1820	2667	3151	4240	4845	6176	6902	8475	9322	11137
4	7	$169U_k+7$	1190	2035	2542	3725	4401	5922	6767	8626	9640	11837	13020	1551	16907
5	12	$289U_k+12$	3480	4377	6370	7526	10127	11572	14751	16485	20242	22265	26600	28912	33825
6	15	$361U_k+15$	5430	7957	9401	12650	14455	18426	20592	25285	27812	33227	36115	42252	45501
7	22	$529U_k+22$	11660	13776	18537	21182	27001	30175	37052	40755	48690	52922	61915	66676	76727
8	26	$625U_k$+26	16272	21901	25026	31901	35651	43776	48151	57526	62526	73151	78776	90651	96901
9	35	$841U_k+35$	29470	33675	42926	47972	58905	64792	77407	84135	98432	106001	121980	130390	148051
10	40	$961U_k+40$	38480	49051	54817	67310	76037	88452	96140	112477	121126	139385	148995	169176	179747
11	51	$1225U_k+51$	62526	69876	85800	94376	103551	122551	143376	154401	177676	189926	215651	229126	257301
12	57	$1369U_k+57$	78090	95887	105470	126005	136957	160230	172551	198562	212252	241001	256060	287547	303975
13	70	$1681U_k+70$	117740	129507	154722	168170	196747	211876	243815	260625	295926	314417	353080	373252	415277
14	77	$1849U_k+77$	142373	170185	184977	216410	233051	268182	286682	325501	345840	388367	410555	456780	480817
15	92	$2209U_k+92$	203230	220992	258545	278426	320397	342487	388876	413175	463982	490490	545715	574432	634075
16	100	$2401U_k+100$	240200	281017	302626	348345	372255	422676	449087	504310	533122	593147	624360	689187	722801
17	117	$2809U_k+117$	328770	354051	410231	435512	494501	525400	590007	623715	693940	730457	806300	845626	927087
18	126	$3025U_k+126$	381276	438751	469001	532526	565801	635376	671676	747301	786626	868301	910651	998376	1043751
19	145	$3481U_k+145$	504890	539700	612801	651092	731155	772927	859952	905205	999192	1047926	1148875	1201090	1309001
20	155	$3721U_k+155$	576755	655051	695982	781565	826217	919242	967615	1068082	1120176	1228085	1283900	1399251	1458787
21	176	$4225U_k+176$	743776	790251	887426	938126	1043751	1098676	1212751	1271901	1394426	1457801	1588776	1656376	1795801
22	187	$4489U_k+187$	839630	942877	996745	1108970	1167327	1288530	1351376	1481557	1548892	1688051	1759875	1908012	1984325
23	210	$5041U_k+210$	1058820	1119312	1245337	1310870	1446977	1517551	1663740	1739355	1895626	1976282	2142635	2228332	2404767
24	222	$5329U_k+222$	1183260	1316485	1385762	1529645	1604251	1758792	1838727	2003926	2089190	2265047	2355640	2542155	2638077
25	247	$5929U_k+247$	1464710	1541787	1701870	1784876	1956817	2045752	2229551	2324415	2520072	2620865	2828380	2935102	3172262
26	260	$6241U_k+260$	1622920	1791427	1878801	2059790	2153405	2346876	2446732	2652685	2758782	2977217	3089555	3339195	3439051
27	287	$6889U_k+287$	1977430	2073876	2273657	2376992	2590551	2700775	2928112	3045225	3286340	3410342	3685902	3796126	4064797
28	301	$7225U_k+301$	2175026	2384551	2492926	2716901	2832501	3070926	3193751	3229876	3576676	3865676	3981276	4263051	4407551
29	330	$7921U_k+330$	2614231	2733075	2978626	3105362	3366755	1501412	3778647	3921225	4238065	4364801	4673720	4832140	5156901
30	345	$8281U_k+345$	2857290	3114001	3246497	3519770	3662547	3950382	4099440	4377715	4563176	4886135	5051755	5391276	5921260

In this table,$25U_{k+1}$ in the first place is started from 1, $49U_k+2$ in the second place is started from 2, and the term $121U_k+5$ i in the third place is started from 5. Then we find $U_k = U_q$ without prime numbers. As a result, we find out which U_k gives prime numbers, or more precisely how we find U_k without prime numbers and the reason why they have no prime numbers. As it is easy to create Table 7, finding $U_(q)$ without prime numbers is a simple method. If a computer program is written, we can calculate a U_q that is to a specified U_k value and that gives composite numbers. $(U_k \geq U_q)$ If the U_q that is found this way is taken out from the general U_k, we find out the U_p. In Table 7, we see that $U_k = 1276$ value passing through both columns of 7 and 11. That is the reason of it following:

$$25.51+1 = 1276 \qquad 49.26+2 = 1276$$

$$U_k = 1276 \qquad Q_k = 175 = 5.35 = 7.25$$

As we can see, two time repetition of number with three times, four times, five times of U_k members will occur. Yet, wisely prepared Computer programming software can elect the repetitions from the list.

2.1 Some Properties of U_n(Pentagonal Numbers)

The sequence U_n has many features that are far more than I could find. I am sure that a curious reader will find some interesting ones.

1.

$$U_5 - U_1 = Q_3$$
$$U_6 - U_2 = Q_4$$
$$U_7 - U_3 = Q_5$$
$$U_8 - U_4 = Q_6$$
$$U_9 - U_5 = Q_7$$
$$U_{10} - U_6 = Q_8$$

****

$$U_{k-2} - U_{k-6} = Q_{k-4}$$
$$U_{k-1} - U_{k-5} = Q_{k-3}$$
$$U_k - U_{k-4} \quad = Q_{k-2}$$
$$U_{k+1} - U_{k-3} = Q_{k-1}$$
$$U_{k+2} - U_{k-2} = Q_k$$
$$U_{k+3} - U_{k-1} = Q_{k+1}$$
$$U_{k+4} - U_k \quad = Q_{k+2}$$
$$\sum_{i=1}^{4} U_{k+i} - \sum_{i=1}^{4} U_i = Q_3 + Q_4 \cdots + Q_{k-1} + Q_k + Q_{k+1} + Q_{k+2}$$
$$\sum_{i=1}^{4} U_{k+i} = \sum_{i=1}^{4} U_i + \sum_{k=1}^{\infty} Q_{k+2} - Q_1 - Q_2$$
$$\sum_{i=1}^{4} U_{k+i} = 1 + 2 + 5 + 7 + \sum_{k=1}^{\infty} Q_{k+2} - 5 - 7$$
$$\sum_{k=1}^{\infty} Q_{k+2} + 3 = \sum_{i=1}^{4} U_{k+i} \tag{2.7}$$

2.

$$U_n = \frac{3n^2 - n}{2} \qquad U_{n+1} = \frac{3n^2 + n}{2}$$
$$U_{n+2} = \frac{3(n+1)^2 - (n+1)}{2} \qquad U_{n+3} = \frac{3(n+1)^2 + (n+1)}{2}$$
$$U_{n+4} = \frac{3(n+2)^2 - (n+2)}{2} \qquad U_{n+5} = \frac{3(n+2)^2 + (n+2)}{2}$$
$$U_{n+6} = \frac{3(n+3)^2 - (n+3)}{2} \qquad U_{n+7} = \frac{3(n+3)^2 + (n+3)}{2}$$

After these equations are simplified, we find the recursive sequence.

$$U_{n+1} = U_n + n$$

$$U_{n+3} = U_{n+2} + n + 1$$

$$U_{n+5} = U_{n+4} + n + 2$$

$$U_{n+7} = U_{n+6} + n + 3$$

$$U_{n+7} = U_{n+6} + U_{n+5} - U_{n+4} + U_{n+3} - U_{n+1} + U_n \tag{2.8}$$

$$U_{4n} - U_{4n-1} - U_{4n-2} + U_{4n-3} + U_{4n-5} - U_{4n-6} \cdots + U_1 = n;\ (2n)$$

k=odd: n k=even: 2n

3.

$$k.Q_{2k} = U_{4k} \quad k = \frac{U_{4k}}{Q_{2k}} \tag{2.9}$$

$$k.Q_{2k} = U_{4k}$$

$$k.Q_{2k-1} = U_{4k-1}$$
$$\frac{Q_{2k}}{Q_{2k-1}} = \frac{U_{4k}}{U_{4k-1}} \tag{2.9a}$$

2.2 Studying The Quantities of Q_k Numbers $[\lambda(Q_k^2)]$

Table 8

k	U_n	Q_n	k	U_n	Q_n
1	1	5	29	330	89
2	2	7	30	345	91
3	5	11	31	376	95
4	7	13	32	392	97
5	12	17	33	425	101
6	15	19	34	442	103
7	22	23	35	477	107
8	26	25	36	495	109
9	35	29	37	532	113
10	40	31	38	551	115
11	51	35	39	590	119
12	57	37	40	610	121
13	70	41	41	651	125
14	77	43	42	672	127
15	92	47	43	715	131
16	100	49	44	737	133
17	117	53	45	782	137
18	126	55	46	805	139
19	145	59	47	852	143
20	155	61	48	876	145
21	176	65	49	925	149
22	187	67	50	950	151
23	210	71	51	1001	155
24	222	73	52	1027	157
25	247	77	53	1080	161
26	260	79	54	1107	163
27	287	83	55	1162	167
28	301	85	56	1190	169

We start our study by an example: (Table 8)

1. $k = 4$ $U_4 = 7$ $Q_4 = 13$

   $Q_4^2 = 169$; 169 When there is a new Q_k number, its value is 56. If we pay attention $56 = 8.7 = 8U_4$; The number 56 is the quantity of all Q_k numbers until 169. We may show this as $\lambda(Q_4^2) = 56$

   At the same time $\lambda(Q_4^2) = 4(13+1) = 56$ ($k = 4$ even number)

2. $k = 5$ $U_5 = 12$ $Q_5 = 17$

   $Q_5^2 = 289$ $8U_5 = 8.12 = 96$

   $\lambda(Q_5^2) = 96$ $\lambda(Q_5^2) = 5(17+2)+1 = 96$ (k=5 odd number)

   We can multiply these examples. Generalizing:

   $\lambda(Q_k^2) = k(Q_k+1)$ $k =$ çift sayı

   $\lambda(Q_k^2) = k(Q_k+2)+1$ $k =$ tek sayı

   More examples:

   $k = 20$ $U_{20} = 61$ $Q_{20} = 15$

   $\lambda(Q_{20}^2) = 20(61+1) = 1240$ $8.155 = 1240$

   $k = 37$ $U_{37} = 113$ $Q_{37} = 532$

   $\lambda(Q_{37}^2) = 37(113+2) = 4256$ $8.532 = 4256$

$n = 1$ $k_1 = 2n-1 = 1$

$n = 1$ $k_2 = 2n = 2$

$n = 2$ $k_3 = 2n-1 = 3$

$n = 2$ $k_4 = 2n = 4$

k is odd: $n = \dfrac{k+1}{2}$ $Q_n = 6n-1$ $Q_k = 3k+2$

k is even: $n = \dfrac{k}{2}$ $Q_n = 6n+1$ $Q_k = 3k+1$

$\lambda(Q_k^2) = k(Q_k+2)+1 = k(3k+4)+1 = 3k^2+4k+1$

$\lambda(Q_k^2) = (3k+1)(k+1)$ $k+1 = 2n$ and $3k+1 = 6n-2$

$\lambda(Q_k^2) = 2n(6n-2) = 12n^2-4n = 8.\dfrac{3n^2-n}{2}$

$\lambda(Q_k^2) = 8U_n$

Using the same method, for the even k

$\lambda(Q_k^2) = k(Q_k+1) = k(3k+2)$ $k = 2n$ $3k+2 = 6n+2$

$\lambda(Q_k^2) = 2n(6n+2) = 12n^2+4n = 8.\dfrac{3n^2+n}{2}$

$\lambda(Q_k^2) = 8U_n$ is found.

To sum up, The number of all terms until Q_n^2 (Q_n^2 included) equals $8U_n$

* * *

The terms less than Q_k

$Q_1, Q_2, Q_3, \ldots\ldots\ldots\ldots\ldots\ldots Q_{k-1}$ respectively.

If the Q_k multiplicand of Q_1 which approximates a maximum (Q_k^2) is Q_{y_1}. This is Q_{y_2} for Q_2 and $Q_{y_{k-1}}$ for Q_{k-1}.

Q_1	Q_{y_1}	$Q_1Q_{y_1} < Q_k^2$
Q_2	Q_{y_2}	$Q_2Q_{y_2} < Q_k^2$
Q_3	Q_{y_3}	$Q_3Q_{y_3} < Q_k^2$
....	....	....
Q_{k-1}	$Q_{y_{k-1}}$	$Q_{k-1}Q_{y_{k-1}} < Q_k^2$
Q_k	Q_{y_k}	$Q_kQ_{y_k} = Q_k^2$

Example 1

$Q_5 = 17,$ $Q_5^2 = 289$

$Q_1 = 5$	$Q_{y_1} = 55$	$55 = 3.18+1$
$Q_2 = 7$	$Q_{y_2} = 41$	$41 = 3.13+2$
$Q_3 = 11$	$Q_{y_3} = 25$	$25 = 3.8+1$
$Q_4 = 13$	$Q_{y_4} = 19$	$19 = 3.6+1$
$Q_5 = 17$	$Q_{y_5} = 17$	$17 = 3.5+2$

Example 2

$Q_6 = 19, \qquad Q_6^2 = 361$

$Q_1 = 5$	$Q_{y_1} = 71$	$71 = 3.23 + 2$
$Q_2 = 7$	$Q_{y_2} = 49$	$49 = 3.16 + 1$
$Q_3 = 11$	$Q_{y_3} = 31$	$31 = 3.10 + 1$
$Q_4 = 13$	$Q_{y_4} = 25$	$25 = 3.8 + 1$
$Q_5 = 17$	$Q_{y_5} = 19$	$18 = 3.6 + 1$
$Q_6 = 19$	$Q_{y_6} = 19$	$18 = 3.6 + 1$

Example 3

$Q_{11} = 35, \qquad Q_{11}^2 = 1225$

$Q_1 = 5$	$Q_{y1} = 245$	$254 = 3.81 + 2$
$Q_2 = 7$	$Q_{y2} = 175$	$175 = 3.36 + 1$
$Q_3 = 11$	$Q_{y3} = 109$	$109 = 3.29 + 2$
$Q_4 = 13$	$Q_{y4} = 89$	$89 = 3.29 + 1$
$Q_5 = 17$	$Q_{y5} = 71$	$71 = 3.23 + 2$
$Q_6 = 19$	$Q_{y6} = 61$	$61 = 3.20 + 1$
$Q_7 = 23$	$Q_{y7} = 53$	$53 = 3.17 + 1$
$Q_8 = 25$	$Q_{y8} = 49$	$49 = 3.16 + 1$
$Q_9 = 29$	$Q_{y9} = 41$	$41 = 3.13 + 2$
$Q_{10} = 31$	$Q_{y10} = 37$	$37 = 3.12 + 1$
$Q_{11} = 35$	$Q_{y11} = 35$	$35 = 3.11 + 2$

Example 4

$Q_{14} = 43, \qquad Q_{14}^2 = 1849$

$Q_1 = 5$	$Q_{y1} = 365$	$365 = 3.121 + 2$
$Q_2 = 7$	$Q_{y2} = 263$	$263 = 3.87 + 2$
$Q_3 = 11$	$Q_{y3} = 167$	$167 = 3.55 + 2$
$Q_4 = 13$	$Q_{y4} = 139$	$139 = 3.46 + 1$
$Q_5 = 17$	$Q_{y5} = 107$	$107 = 3.35 + 2$
$Q_6 = 19$	$Q_{y6} = 97$	$97 = 3.32 + 1$
$Q_7 = 23$	$Q_{y7} = 79$	$79 = 3.26 + 1$
$Q_8 = 25$	$Q_{y8} = 73$	$73 = 3.24 + 1$
$Q_9 = 29$	$Q_{y9} = 61$	$61 = 3.20 + 1$
$Q_{10} = 31$	$Q_{y10} = 59$	$59 = 3.19 + 1$
$Q_{11} = 35$	$Q_{y11} = 49$	$49 = 3.16 + 1$
$Q_{12} = 37$	$Q_{y12} = 49$	$49 = 3.16 + 1$
$Q_{13} = 41$	$Q_{y13} = 43$	$43 = 3.14 + 1$
$Q_{14} = 43$	$Q_{y14} = 43$	$43 = 3.14 + 1$

We have given examples for $Q_5 = 17$,$Q_6 = 19$,$Q_{11} = 35$ and $Q_{14} = 43$ Notice that as all Q_y are odd number; if the quotient is an odd number, we add 2; and if the quotient is an even number, we add 1.

The most significant feature in these tables:

We rewrite the quotients, when Q_y is divided by 3. For instance we select $Q_5 = 17$ and $Q_6 = 19$

$Q_5 = 17$	C1	$Q_6 = 19$	C2
$Q_1 = 5$	18	$Q_1 = 5$	23
$Q_2 = 7$	13	$Q_2 = 7$	16
$Q_3 = 11$	8	$Q_3 = 11$	10
$Q_4 = 13$	6	$Q_4 = 13$	8
$Q_5 = 17$	5	$Q_5 = 17$	6
		$Q_6 = 19$	6

In the section $Q_5 = 17$, the meaning of the number 18 corresponding to $Q_1 = 5$: The number $Q_1 = 5$ is multiplied by 18 various Q_k to $(Q_5)^2 = 289$ and a composite number is obtained. The number of resulting composite numbers are 13 when multiplied by 7, 8 when multiplied by 11, 6 when multiplied by 13, and 5 when multiplied by 17. The numbers in the column C1 or C2 give us the quantity of the composite numbers. Especially, when we consider the sum of the quantities of the composite, we should pay attention to one more thing.

Example 5:

$Q_7 = 23$, $Q_7^2 = 529$

$Q_1 = 5$	103	3.34+1	$C3$
$Q_2 = 7$	73	3.24+1	1
$Q_3 = 11$	47	3.15+2	2
$Q_4 = 13$	37	3.12+1	3
$Q_5 = 17$	31	3.10+1	4
$Q_6 = 19$	25	3.8+1	5
$Q_7 = 23$	23	3.7+2	6

The number 5 is multiplied by all of the numbers $[Q_1, Q_2, Q_3, Q_4, Q_5, Q_6,]$ to Q_7. Then, when the number 7 is multiplied by $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6$. In this way, $5x7$ has been multiplied twice. Again, the number $Q_7 = 23$ at the bottom is multiplied by the following numbers 6 times. The numbers in the column C3 give us the quantity of the products. Calculating the sum: $\frac{6.7}{2} = 21$ The quantity of the composite numbers: $34+24+15+12+10+8+7 = 110$

The expected quantity: $110 - 21 = 89$

There are repetitive composite numbers among these

Example:

$7.25 = 5.35$; $7.35 = 5.49$

We do not know the number of these. We indicate the quantities of the numbers by λ

If we use the symbol M_k for each $\lambda(M_k)$, for example $\lambda(M_5) = 3$

Respectively

Table 9

$13 : \lambda(M_4) = 0$	$31 : \lambda(M_{11}) = 51$
$17 : \lambda(M_5) = 3$	$37 : \lambda(M_{12}) = 58$
$19 : \lambda(M_6) = 4$	$41 : \lambda(M_{13}) = 88$
$23 : \lambda(M_7) = 10$	$43 : \lambda(M_{14}) = 99$
$25 : \lambda(M_8) = 16$	$47 : \lambda(M_{15}) = 131$
$29 : \lambda(M_9) = 24$	$49 : \lambda(M_{16}) = 153$
$31 : \lambda(M_{10}) = 34$	$53 : \lambda(M_{17}) = 177$

$\lambda(Q_k)$ shows shows less than or equal to Q_k^2. Then $Q_k.Q_{y_k} \leq Q_k^2$ and Q_{y_k} is the nearest term to Q_k^2.

$$Q_{y_1} = 3\lambda(Q_1) + 1(2)$$

$$Q_{y_2} = 3\lambda(Q_2) + 1(2)$$

$$\dots$$

$$Q_{y_{k-1}} = 3\lambda(Q_{k-1}) + 1(2)$$

$$Q_{y_k} = 3\lambda(Q_k) + 1(2)$$

Here, the sum of the $\lambda(Q_k)$'s gives us the gross composite numbers. For the net number, we should take both $\frac{k(k-1)}{2}$ and $\lambda(M_k)$ out.

If the quantity of the composite numbers to Q_k^2: $\lambda(q)$

And the quantity of the prime numbers to Q_k^2: $\lambda(p)$

$\lambda(q) = \sum \lambda(Q_k) - \frac{k(k-1)}{2} - \lambda(M_k)$ The total quantity of numbers to Q_k^2 is $8U_k$.

$8U_k = \lambda(p) + \lambda(q)$ and $Q_k = \sqrt{24U_k + 1}$

Before we conclude:

If $Q_k - Q_{k-1} = 2$ is the last term $Q_{y_{k-1}}$

$$Q_k = Q_{y_{k-1}}$$

If $Q_k - Q_{k-1} = 4$ is the last term $Q_{y_{k+1}}$

$$Q_k = Q_{y_k} \ \textit{and} \ Q_k = Q_{y_{k+1}}$$

Further studies may be conducted on this subject. Especially, do the $\lambda(M)$ numbers have an order? Are these numbers also irregular like the distribution of the prime numbers? My study on this subject is limited.

Chapter 3

GEOMETRIC ANALYSIS

A sequence $Q_1, Q_2, Q_3, ..., Q_n$ with no multiples of 2 and 3 is assumed as a term Q_r. Q_r According to the formula (2.1), the element generating the term is U_r.

$Q_r = \sqrt{24U_r + 1}$.

We see that the number of terms till is Q_r^2. Among these terms, there are prime numbers, relatively prime numbers and relative products. We indicate prime numbers as p and other numbers/composite numbers as q.

$\lambda(p)$: The number of prime numbers

$\lambda(q)$: The number of composite numbers

$$\lambda(p) + \lambda(q) = 8U_n \tag{3.10}$$

For every Q_n, there are $\lambda(p)$ and $\lambda(q)$ and their sum remains constant

On the other hand, for every n, there are two U_n 's. For $n = 1$, $U_{n_1} = 1$ and $U_{n_2} = 2$ can be given as example.

$n:$		1		2		3		4		5		6
$U_n:$	1,	2,	5,	7,	12,	15,	22,	26,	35,	40,	51,	57
$Q_n:$	5,	7,	11,	13,	17,	19,	23,	25,	29,	31,	35,	37
		2	4	2	4	2	4	2	4	2	4	2

The sequence Q_n increases according to the row 2 and 4. The first term plus 2 generates the second term, second term plus 4 generates third term. Here, because of this course, we do not see a regular increase between two terms. As we want to handle the quantity of prime numbers, if we choose the second terms, a term differs from another term with a constant increase of $2+4=6$.

As we will not deal with the firs term after that, the n row now involves the second term of the sequence U_n. Please remember that we do not include the prime numbers 2 and 3 into these sums.

Table 10

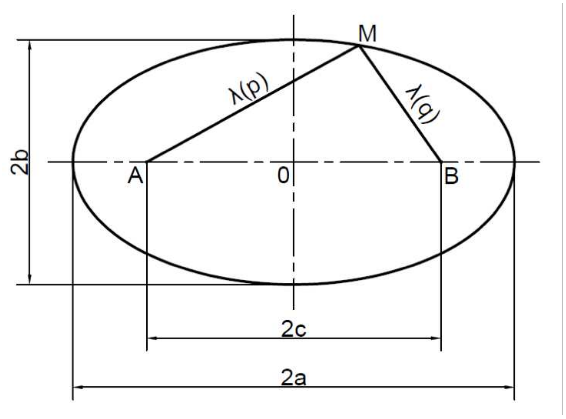
n	$8U_n$	$\lambda(p)$	$\lambda(q)$	n	$8U_n$	$\lambda(p)$	$\lambda(q)$
1	16	13	3	11	1496	607	839
2	56	37	19	12	1776	703	1073
3	120	70	50	13	2080	809	1271
4	208	112	96	14	2408	921	1487
5	320	160	160	15	2760	1036	1724
6	456	217	239	16	3106	1161	1975
7	616	281	335	17	3536	1292	2244
9	800	355	445	18	3960	1421	2539
10	1008	432	576	19	4408	1570	2838
11	1240	517	723	20	4880	1713	3167

Paid attention to the table, till, $\lambda(p) > \lambda(q)$ and for $n=5$ $\lambda(p)=\lambda(q)$. Hereafter, the table proceeds in a way that $\lambda(q) > \lambda(p)$. $\lambda(p)$ and $\lambda(q)$ changes according to U_n. but their sum remains constant.

This reminds me of the locus of the lines which have constant sums of the distances to the focal two points. As we know, this is the geometric figure is called as 'ellipse' .

The major axis of this ellipse is $2a$, the minor axis is $2b$ and the focal point is $2c$.

$$\lambda(p) + \lambda(q) = 8U_n = 2a$$



We may give numerical examples of the ellipse equations:

Table 11

$n=1$	U_1=2	$8U_1$=16	M_1	16=13 + 3
$n=2$	U_2=7	$8U_2$=56	M_2	56=37 + 19
$n=3$	U_3=15	$8U_3$=120	M_3	129=70 + 50
$n=4$	U_4=26	$8U_4$=208	M_4	208=112 + 96
$n=5$	U_5=40	$8U_5$=320	M_5	320=160 + 160
$n=6$	U_6=57	$8U_6$=456	M_6	456=217 + 239

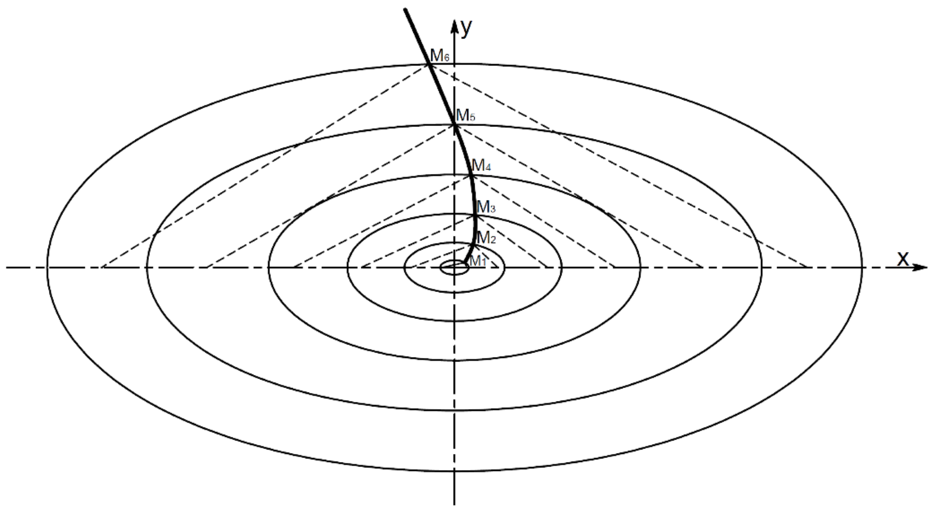


Fig. 3.2

We mark the points $M_1, M_2, ..., M_6$, in accordance with these values. Matching these points, we find the locus of the point M. This curve progresses to the decreasing part of the axis x. Commonly, as it is easy to draw curves according to the increasing part of the axis x, we may draw it again by placing the major axis of ellipse (2a) over the axis. (Figure 3.3)

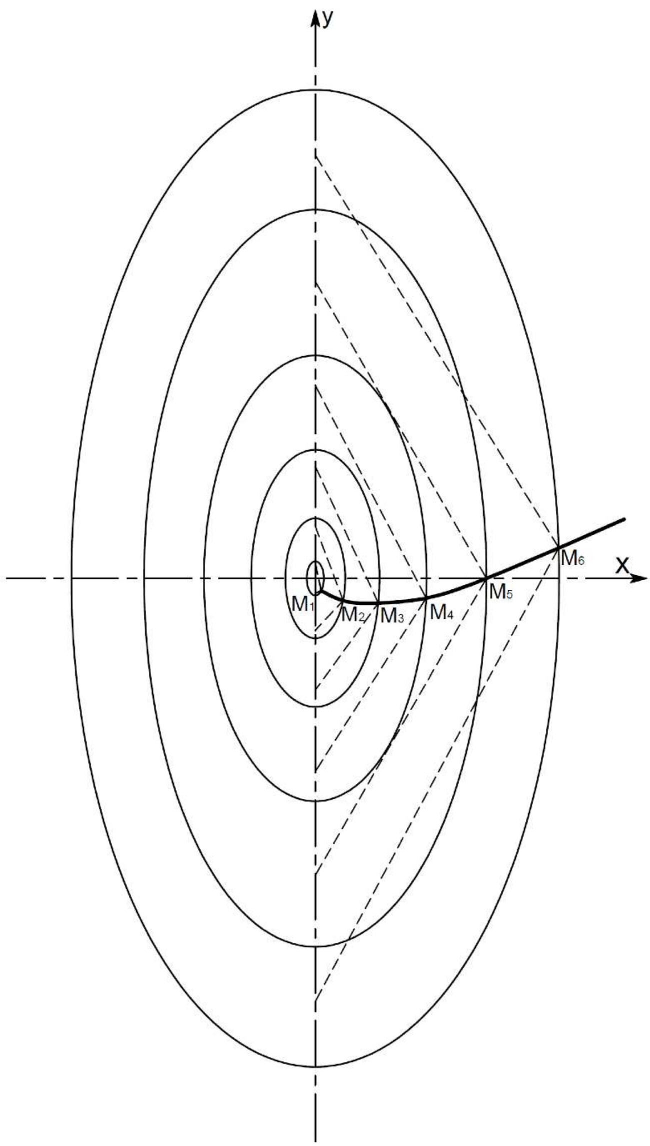


Fig. 3.3

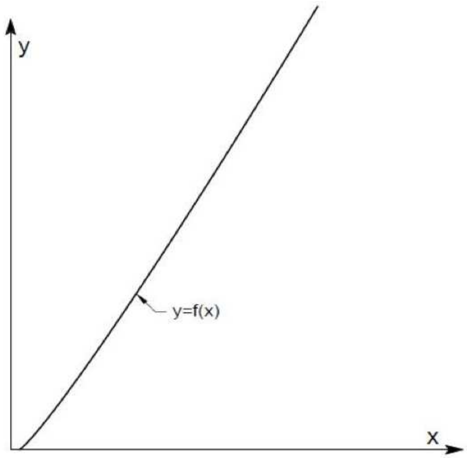


Fig. 3.4

It is the real-like example of the locus curve that is drawn schematically. Now, we may draw any of these ellipses which are greater than :

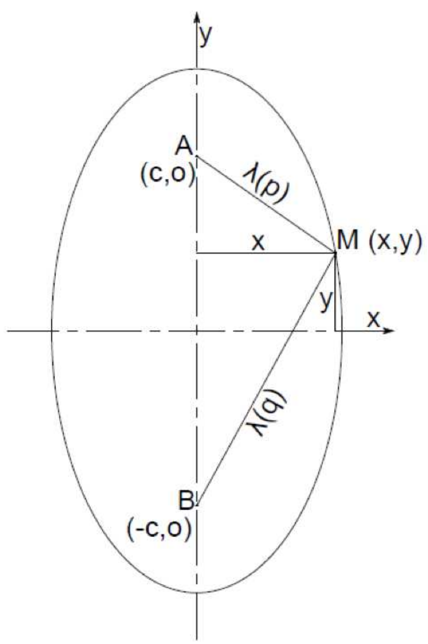


Fig. 3.5

We only know the axis $2a$ of this ellipse. We need to choose the minor axis or the focal point.

Assuming that $1 > mb = m.a$ (3.11)

$$c^2 = a^2 - b^2 = a^2 - m^2a^2$$

$$c = a\sqrt{1-m^2} \tag{3.12}$$

We may find the coordinates of the point $M(x,y)$

$$\overline{MA^2} = \lambda^2(p) = (c-y)^2 + x^2 \qquad \lambda(p) = \sqrt{(c-y)^2 + x^2} \tag{3.12a}$$

$$\overline{MB^2} = \lambda^2(q) = (c+y)^2 + x^2 \qquad \lambda(q) = \sqrt{(c+y)^2 + x^2} \tag{3.12b}$$

$$\lambda^2(q) - \lambda^2(p) = 4cy$$

$$y = \frac{[\lambda(q)-\lambda(p)][\lambda(q)+\lambda(p)]}{4c} = [\lambda(q)-\lambda(p)]\frac{2a}{4c}$$

ve $\quad c = a\sqrt{1-m^2}$

$$y = \frac{[\lambda(q)-\lambda(p)]}{2\sqrt{1-m^2}} \tag{3.13}$$

The equation of the ellipse:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \qquad a^2x^2 + b^2y^2 = a^2b^2 \qquad b = ma$$

$$a^2x^2 + m^2a^2y^2 = m^2a^4 \qquad x^2 + m^2y^2 = 16U_n^2m^2 \tag{3.14}$$

$$x^2 = m^2a^2 - m^2y^2$$

$$x = m\sqrt{a^2 - y^2} \quad \text{or} \tag{3.15}$$

$$x = m\sqrt{16U_n^2 - y^2} \tag{3.16}$$

$$2a = \lambda(p) + \lambda(q) \qquad a = \frac{\lambda(p)+\lambda(q)}{2} \qquad y = \frac{\lambda(q)-\lambda(p)}{2\sqrt{1-m^2}} \tag{3.13}$$

We may replace the values a and y in (3.15):

$$x = m\sqrt{\frac{[\lambda(p)+\lambda(q)]^2}{4} - \frac{[\lambda(q)-\lambda(p)]^2}{4(1-m^2)}}$$
$$x = \frac{m}{2}\sqrt{\frac{[\lambda(p)+\lambda(q)]^2(1-m^2)-[\lambda(q)-\lambda(q)]^2}{1-m^2}}$$
$$x = \frac{m}{\sqrt{1-m^2}}\sqrt{\lambda(p)\lambda(q) - m^2\left(\frac{\lambda(p)+\lambda(q)}{2}\right)^2}$$
$$x = \frac{m}{2\sqrt{1-m^2}}\sqrt{4\lambda(p)\lambda(q) - m^2[\lambda(p)+\lambda(q)]^2} \tag{3.17}$$

$$\frac{(1-m^2)x^2}{m^2} = \lambda(p)\lambda(q) - m^2\left(\frac{\lambda(p)+\lambda(q)}{2}\right)^2$$
$$\left(a = 4U_n = \frac{\lambda(p)+\lambda(q)}{2}\right)$$

$$x^2 + m^2y^2 = m^2\left(\frac{\lambda(p)+\lambda(q)}{2}\right)^2 \tag{3.14a}$$

$$\frac{(1-m^2)x^2}{m^2} + x^2 + m^2y^2 = \lambda(p)\lambda(q)$$

$$x^2 + m^4y^2 = m^2\lambda(p)\lambda(q) \tag{3.14b}$$

$$\lambda(p)\lambda(q) = \frac{x^2+m^4y^2}{m^2} \qquad \lambda(p)+\lambda(q) = \frac{2\sqrt{x^2+m^2y^2}}{m}$$

$$w^2 - Sw + P = 0$$
$$w^2 - \frac{2\sqrt{x^2+m^2y^2}}{m}w + \frac{x^2+m^4y^2}{m^2} = 0$$

$$\lambda(q) = \frac{1}{m}\sqrt{x^2+m^2y^2} + y\sqrt{1-m^2}$$
$$\lambda(p) = \frac{1}{m}\sqrt{x^2+m^2y^2} - y\sqrt{1-m^2}$$
$$\lambda(q) = 4U_n + y\sqrt{1-m^2} \tag{3.14c}$$
$$\lambda(p) = 4U_n - y\sqrt{1-m^2} = \sqrt{\frac{(1-m^2)}{m^2}(16U_n^2m^2 - x^2)} \tag{3.14d}$$

$$\lambda(p)\lambda(q) = 16U_n^2 - y^2(1-m^2) \tag{3.14e}$$

The equation of Ellipse: $x^2 + m^2y^2 = m^2a^2$

Assume that $t = \dfrac{y}{x}$ and $r = \dfrac{\lambda(q)}{\lambda(p)}$

$$\frac{y^2}{t^2} + m^2y^2 = m^2a^2y^2 = t^2m^2(a^2 - y^2) \tag{3.18}$$

$$y = \frac{\lambda(q) - \lambda(p)}{2\sqrt{1-m^2}} \qquad y^2 = \frac{[\lambda(q) - \lambda(p)]^2}{4(1-m^2)}$$

$$a = \frac{\lambda(q) + \lambda(q)}{2} \qquad a^2 = \frac{[\lambda(q) + \lambda(1)]^2}{4}$$

If we replace these values above:

$$\frac{[\lambda(q) - \lambda(p)]^2}{4(1-m^2)} = t^2m^2\left[\frac{[\lambda(q) + \lambda(p)]^2}{4} - \frac{[\lambda(q) - \lambda(p)]^2}{4(1-m^2)}\right] \tag{3.19}$$

$$\frac{[\lambda(q) + \lambda(p)]^2}{[\lambda(q) - \lambda(p)]^2}(1-m^2) = \frac{1}{m^2t^2} + 1 \tag{3.20}$$

$$\frac{1}{m^2t^2} = \frac{\left[\dfrac{\lambda(q)}{\lambda(p)} + 1\right]^2}{\left[\frac{\lambda(q)}{\lambda(p)} - 1\right]^2}(1-m^2) - 1$$

$$\frac{1}{m^2t^2} = \frac{4\dfrac{\lambda(q)}{\lambda(p)} - m^2\left[\dfrac{\lambda(q)}{\lambda(p)} + 1\right]^2}{\left[\dfrac{\lambda(q)}{\lambda(p)} - 1\right]^2} = \frac{4r - m^2(r+1)^2}{(r-1)^2}$$

$$t = \frac{r-1}{m\sqrt{4r - m^2(r+1)^2}} \tag{3.21}$$

$$4r - m^2(r+1)^2 > 0 \qquad 0 > m^2r^2 - 2r(2-m^2) + m^2$$

For the limit values $m^2r^2 - 2r(2-m^2) + m^2 = 0$ (3.21a)

$$r = \frac{2 - m^2 \pm 2\sqrt{1-m^2}}{m^2} = \frac{(1 \pm \sqrt{1-m^2})^2}{m^2}$$

$$\left(\frac{1+\sqrt{1-m^2}}{m}\right)^2 > r > \left(\frac{1-\sqrt{1-m^2}}{m}\right)^2 \text{ or} \tag{3.21c}$$

$$\left(\frac{1+\sqrt{1-m^2}}{m}\right)^2 > r > \frac{1}{\left(\dfrac{1+\sqrt{1-m^2}}{m}\right)^2}$$

We may examine the state when the denominator of the equation (3.21) is greater than 1 $(n \geq 10)$

$$\sqrt{4\frac{\lambda(q)}{\lambda(p)} - m^2\left[\frac{\lambda(q)}{\lambda(p)}+1\right]^2} = \sqrt{4r - m^2(r+1)^2} > 1$$
$$4r - m^2(r+1)^2 > \frac{1}{m^2}, \qquad 0 > m^4r^2 - 2rm^2(2-m^2) + m^4 + 1$$

Limit Values

$$\frac{2-m^2+\sqrt{3-4m^2}}{m^2} > r > \frac{2-m^2-\sqrt{3-4m^2}}{m^2} \tag{3.21b}$$

If (r) is among these values,the denominator of the equation (3.21) is greater than 1. We may handle the positive part of this function. There is no extremum point in the positive area. The locus of asymptotes:

$$r_{min} = \left(\frac{1-\sqrt{1-m^2}}{m}\right)^2 \quad \text{and} \quad r_{max} = \left(\frac{1+\sqrt{1-m^2}}{m}\right)^2 \tag{3.21d}$$

For r=1, t=0. We may examine the intercept point of the 45° line –which intercepts this point- with the curve. (The true $t = r - 1$)

$$r - 1 = \frac{r-1}{m\sqrt{4r - m^2(r+1)^2}}$$
$$m^2[4r - m^2(r+1)^2] = 1 \qquad m^4r^2 - 2rm^2(2-m^2) + m^4 + 1 = 0$$
$$r_1 = \frac{2-m^2-\sqrt{3-4m^2}}{m^2} \qquad r_2 = \frac{2-m^2+\sqrt{3-4m^2}}{m^2}$$

We may find the ordinates

$$t_1 = \frac{2-m^2-\sqrt{3-4m^2}}{m^2} - 1 = \frac{2-2m^2-\sqrt{3-4m^2}}{m^2}$$
$$t_2 = \frac{2-2m^2+\sqrt{3-4m^2}}{m^2}$$

The coordinates we found: $A(1,0)$; $A_1(r_1,t_1)$; $B(r_2,t_2)$

r	$\left(\frac{1-\sqrt{1-m^2}}{m}\right)^2$	1	$\frac{2-m^2-\sqrt{3-4m^2}}{m^2}$	$\frac{2-m^2+\sqrt{3-4m^2}}{m^2}$	$\left(\frac{1+\sqrt{1-m^2}}{m}\right)^2$
t	$-\infty$	0	$\frac{2-2m^2-\sqrt{3-4m^2}}{m^2}$	$\frac{2-2m^2+\sqrt{3-4m^2}}{m^2}$	$+\infty$

The drawing of this curve is presented in Figure 3.6.

Point D and C: $r_D = \dfrac{2-m^2-\sqrt{3-4m^2}}{m^2}$; $\quad r_C = \dfrac{2-m^2+\sqrt{3-4m^2}}{m^2}$

$\lambda(q)_C + \lambda(p)_C = 8U_C$

$\lambda(q)_D + \lambda(p)_D = 8U_D$ ve $r_D = \dfrac{\lambda(q)_D}{\lambda(p)_D}$; $r_C = \dfrac{\lambda(q)_C}{\lambda(p)_C}$

$\lambda(p)_C + r_C\lambda(p)_C = 8U_C$

$\lambda(p)_D + r_D\lambda(p)_D = 8U_D \quad 1+r_C = \dfrac{8U_C}{\lambda(p)_C}$; $\quad 1+r_D = \dfrac{8U_D}{\lambda(p)_D}$

$$8U_C = \lambda(p)_C(1+r_C) = \lambda(p)_C\left(\frac{2+\sqrt{3-4m^2}}{m^2}\right)$$

$$8U_D = \lambda(p)_D(1+r_D) = \lambda(p)_D\left(\frac{2-\sqrt{3-4m^2}}{m^2}\right)$$

$$64U_CU_D = \frac{1+4m^2}{m^2}\lambda(p)_C\lambda(p)_D \qquad \frac{64U_CU_D}{\lambda(p)_C\lambda(p)_D} = \frac{1+4m^2}{m^2}$$

$$8U_C = \left(\frac{2+\sqrt{3-4m^2}}{m^2}\right)\lambda(p)_C \qquad 8U_D = \left(\frac{2+\sqrt{3-4m^2}}{m^2}\right)\lambda(p)_D$$

$$\frac{8U_C}{\lambda(p)_C} + \frac{8U_D}{\lambda(p)_D} = \frac{4}{m^2} \qquad 4\left(\frac{U_C}{\lambda(p)_C} + \frac{U_D}{\lambda(p)_D}\right)^2 = \frac{1}{m^4}$$

$\dfrac{64U_CU_D}{\lambda(p)_C\lambda(p)_D} = \dfrac{1}{m^4} + \dfrac{4}{m^2}$ If we replace

$\dfrac{64U_CU_D}{\lambda(p)_C\lambda(p)_D} = 4\left(\dfrac{U_C}{\lambda(p)_C} + \dfrac{U_D}{\lambda(p)_D}\right)^2 + \dfrac{8U_c}{\lambda(p)_C} + \dfrac{8U_D}{\lambda(p)_D}$ or

$$\frac{16U_CU_D}{\lambda(p)_C\lambda(p)_D} = \left(\frac{U_C}{\lambda(p)_C} + \frac{U_D}{\lambda(p)_D}\right)^2 + 2\left(\frac{U_C}{\lambda(p)_C} + \frac{U_D}{\lambda(p)_D}\right)$$

$$\boldsymbol{\left(\frac{U_C}{\lambda(p)_C} + \frac{U_D}{\lambda(p)_D} + 1\right)^2 - \frac{16U_CU_D}{\lambda(p)_C\lambda(p)_D} = 1} \tag{3.22}$$

Eventually, we have found a relation which applies for the points C and D, between the quantity of prime numbers independent from m and the pentagonal numbers specifying them $(U_n's)$. (Figure 3.6)

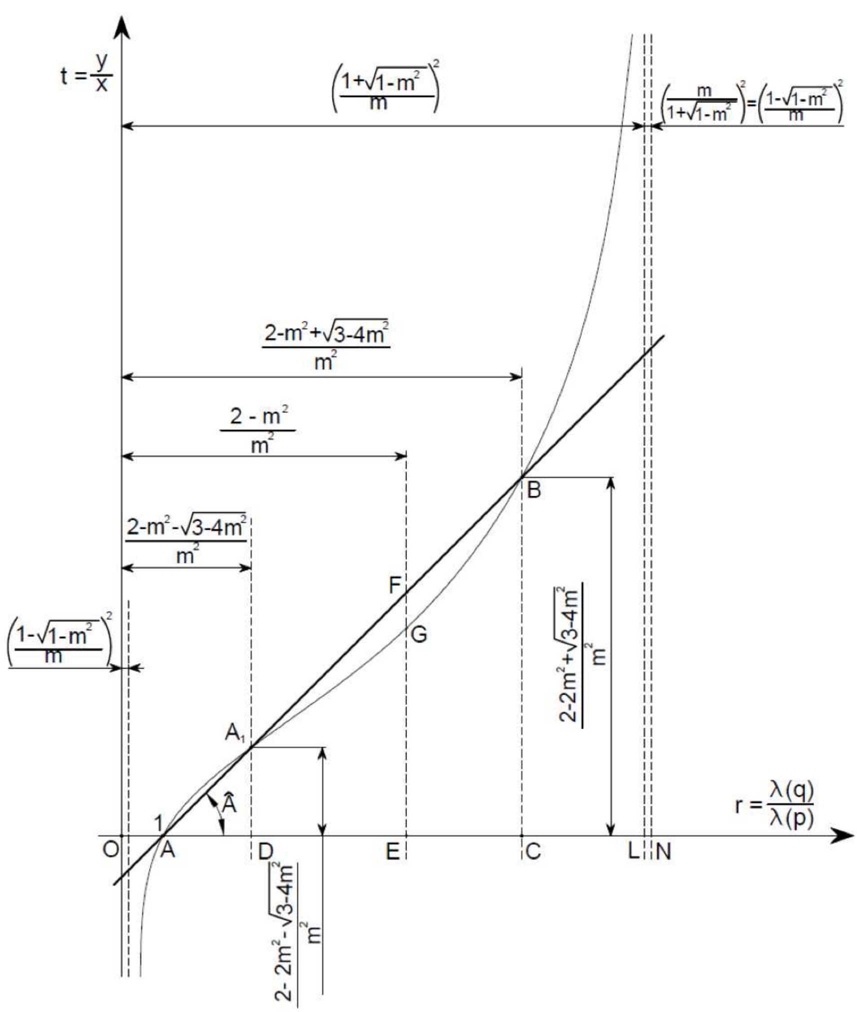


Fig. 3.6

For $m = \frac{1}{2}$ $r_D = 1.343\ldots$ $r_C = 12.656\ldots$

The maximum that I could find on Table A is $r_C = 7.274....$ For this reason, I could not check the formula (3.22) using this list. However, if we calculate m for a certain r, we choose the example and we calculate m, then . In Table A the only without decimal is 1. $r_D = 1$ (n=5)

$$r_D = \frac{2-m^2-\sqrt{3-4m^2}}{m^2} = 1 \qquad 2-2m^2 = \sqrt{3-4m^2}$$

$$4m^4-4m^2+1=0 \qquad (1-2m^2)^2 = 0 \qquad m = \frac{1}{\sqrt{2}}$$

$$r_C = \frac{2-m^2+\sqrt{3-4m^2}}{m^2} \qquad m = \frac{1}{\sqrt{2}} \quad \text{is placed then} \quad r_C = 5$$

$n = 5$ $r_D = 1$ $8U_D = 320$ $\lambda(p)_D = 160$

$n = 2297$ $r_C = 5.0000092856379$ $8U_C = 63323696$ $\lambda(p)_C = 10553933$

If replaced in the formula (3.22): $1.0000000001346 \cong 1$

$m = \frac{1}{\sqrt{2}} = 0.707107...$ Thus, we could check the formula (3.22). Since the expected one of the n values between $n = 2290$ and $n = 2300$ gives closer to integer 5 value, it can be tested the validity of the formula (3.22).

$1+r_C = \frac{8U_C}{\lambda(p)_C}$; $1+r_D = \frac{8U_D}{\lambda(p)_D}$ If we remind these values,and for convenience,if they are replaced in the former formula (3.22) and cancelations:

$$4\sqrt{r_C r_D - 1} = 2 + r_C + r_D \tag{3.21f}$$

On Table 12 ,(page 50) there are numerous r values till $r = 17,848\ldots$. (Note: We should not confuse n, which is the exponent of 10^n with the n we use.) We may choose 10^{16} for example :

Limit values ;$r_L = \frac{\lambda(q)}{\lambda(p)} = 10.937233694309840$

$$r_L = \left(\frac{1+\sqrt{1-m^2}}{m}\right)^2 \qquad m = \frac{2\sqrt{r_L}}{1+r_L} \tag{3.21g}$$

$m = 0.307015350843491$ is found.

$$r_D = \frac{2-m^2-\sqrt{3-4m^2}}{m^2} = 1.178582141749793$$

$$r_C = \frac{2-m^2+\sqrt{3-4m^2}}{m^2} = 9.850082350141047$$

$$\frac{1+r_D}{8} = 0.272322767718724$$

$$\frac{1+r_C}{8} = 1.356260293767630$$ If replaced in (3.21)

$$6.909448911132973 - 5.909448911132960 = 1.000000000000013 \cong 1$$

Table 12

10^n	$\lambda(p)$	$\lambda(q)$	$\frac{\lambda(q)}{\lambda(p)}$
$n=1$	2		
$n=2$	23	7	0.304347826
$n=3$	166	164	0.987951807
$n=4$	1227	2103	1.713936430
$n=5$	9590	23740	2.475495307
$n=6$	78496	254834	3.246458418
$n=7$	664577	2668753	4.015716764
$n=8$	5761453	27571877	4.785577006
$n=9$	50847532	282485798	5.555545901
$n=10$	455052509	2878280821	6.325161962
$n=11$	4118054811	29215278519	7.094436538
$n=12$	37607912016	295725421314	7.863383140
$n=13$	346065536837	2987267796493	8.632086927
$n=14$	3204941750800	30128391582530	9.400605042
$n=15$	29844570422667	303488762910663	10.168977425
$n=16$	279238341033923	3054094992299407	10.937233694
$n=17$	2623557157654231	30709776175679099	11.705396273
$n=18$	24739954287740858	308593379045592472	12.473482184
$n=19$	234057667276344605	3099275666056988725	13.241504549
$n=20$	2220819602560918838	31112513730772414492	14.009473662
$n=21$	21127269486018731926	312206063847314601404	14.777397715
$n=22$	201467286689315906288	3131866046644017427042	15.545283296
$n=23$	1925320391606803968921	31408012941726529364409	16.313135766
$n=24$	18435599767349200867864	314897733565984132465466	17.080959531
$n=25$	176846309399143769411678	3156487023934189563921652	17.848758250

On Table 12 (page 50), there is a formula (3.24) that generates the sum of $[\lambda(p)+\lambda(q)]$ corresponding with the number 10^n. According to this formula, for 10^{16}, $[\lambda(p)+\lambda(q)] = 3333333333333330$ Although this number is not an $8U_n$ (hexagonal number), (It is not correspondent with U_n' s (hexagonal numbers) stated initially.) we prove the accuracy of the formula numerically by assuming it as $8U_n$. It shows us that for the large quantities of prime numbers, there is no problem with calculations made via Table 12. $[\lambda(p)+\lambda(q)]$ In any case, there may be a direcrix of ellipse. However, we expect it to have a value that generates regular sums.

$$8U_n \text{ or } 3\sum_{2}^{n} 10^{n-1}$$

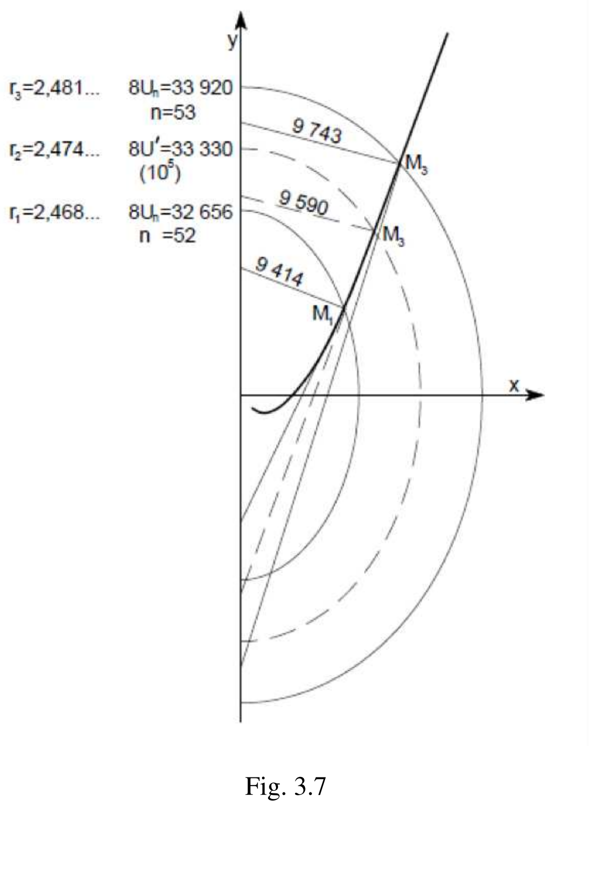


Fig. 3.7

As can be seen in Figure 3.7, it is not wrong to think that, the point M_2 that specifies the value r_2 is somewhere between M_1 and M_3, near or over the locus curve.

If we remove (m) between the abscises of the points E and L in Figure 3.6:

$$2\frac{\lambda(q)_L}{\lambda(p)_L}.\frac{\lambda(q)_E}{\lambda(p)_E}-\frac{\lambda^2(q)_L}{\lambda^2(p)_L}=1 \tag{3.22a}$$

There may such formulas between some prime numbers that are valid in some special points and composite numbers (or pentagonal numbers).

3.1 Choosing the (m) Value

$$t_1 = \frac{2-2m^2-\sqrt{3-4m^2}}{m^2} \qquad \sqrt{3-4m^2} > 0 \qquad \frac{\sqrt{3}}{2} > m \tag{3.17a}$$

On the other hand, from the formula (3.17)

$$x = \frac{m}{2\sqrt{1-m^2}}\sqrt{4\lambda(p)\lambda(q) - m^2[\lambda(p)+\lambda(q)]^2} \tag{3.17}$$
$$4\lambda(p)\lambda(q) > m^2[\lambda(p)+\lambda(q)]^2 \qquad \frac{2\sqrt{\lambda(p)\lambda(q)}}{\lambda(p)+\lambda(q)} > m \tag{3.17b}$$

For $m = \frac{1}{\sqrt{2}}$ $\frac{\sqrt{3}}{2} > \frac{1}{\sqrt{2}}$ and $n = 2297$ $\lambda(p) = 10553933$ and $\lambda(q) = 52769763$ values also on the points D and C on (3.17b) fullfill inequality. However,there is no solution for $\sim n \geq 8000$. ın this formula (3.17) I used the value $m = \frac{1}{\sqrt{2}}$ to check the formula In order to calculate the x,$y$coordinates from the equations (3.13) and (3.17), assuming that $m = \frac{1}{2}$ we calculated (List A).

Approximately till the value $n = 45000$, $x > U_n$

$n = 45000$	$U_n = 3037522500$	$x = 3041750202....$
$n = 46000$	$U_n = 3174023000$	$x = 3172614102....$

Here is $U_n > x$. In this case, somewhere in between $U_n = x$.

If $U_n = x$, I examine the solution that is generated by the formula (3.15)

$x = m\sqrt{a^2 - y^2}$ $\quad a = 4U_n$ $\quad$ and $\quad U_n = x$ $\quad$ olarak:

$U_n = m\sqrt{16U_n^2 - y^2}$

$U_n^2 = m^2(16U_n^2 - y^2)$ In this equation with two unknowns, assuming that $m = \frac{1}{4}$:

$U_n^2 = \frac{1}{16}(16U_n^2 - y^2)$ $\quad 16U_n^2 = 16U_n^2 - y^2$ $\quad y = 0$. This leads us to the one of the initial points in the list

$(n : 5\ x = 40 \quad U_n = 40 \quad$ and $\quad y = 0)$

Now, we may have the coordinates x, y from the equations (3.13) and (3.17) and calculate by assuming that .$m_1 = \frac{1}{2}$ $\quad m_2 = \frac{1}{4}$

$m_1 = \frac{1}{2}$

$x_1 = \frac{1}{2\sqrt{3}}\sqrt{4\lambda(p)\lambda(q) - \left(\frac{\lambda(p)+\lambda(q)}{2}\right)^2}; \quad y_1 = \frac{\lambda(q)-\lambda(p)}{\sqrt{3}}$

$m_2 = \frac{1}{4}$

$x_2 = \frac{1}{2\sqrt{15}}\sqrt{4\lambda(p)\lambda(q) - \left(\frac{\lambda(p)+\lambda(q)}{2}\right)^2}; \quad y_2 = \frac{\lambda(q)-\lambda(p)}{\frac{\sqrt{15}}{2}}$

In the inequality (3.21c) $\left(\frac{1+\sqrt{1-m^2}}{m}\right)^2 > r > \left(\frac{1-\sqrt{1-m^2}}{m}\right)^2$

For $m_1 = \frac{1}{2}$: $\quad \left(2+\sqrt{3}\right)^2 > \frac{\lambda(q)}{\lambda(p)} > \left(2-\sqrt{3}\right)^2$ or

$13.9280 > \frac{\lambda(q)}{\lambda(p)} > 0.071796$

For $m_2 = \frac{1}{4}$: $\quad \left(4+\sqrt{15}\right)^2 > \frac{\lambda(q)}{\lambda(p)} > \left(4-\sqrt{15}\right)^2$ or

$61.9838 > \frac{\lambda(q)}{\lambda(p)} > 0.016133$

We have found the limit values for both cases. However, even if for $m_1 = \frac{1}{2}$ $\frac{\lambda(q)}{\lambda(p)} = 13.9280...$ seems to be the limit value, in the ratios $\frac{\lambda(q)}{\lambda(p)}$ between 10^{19} and 10^{20} a,this value is exceeded. For the quantity of (10^{25}) $\frac{\lambda(q)}{\lambda(p)} = 17.848....$ (Table 12) $13.9280... \geq \frac{\lambda(q)}{\lambda(p)}$ as square root of the formula (3.17) is negative, we cannot the value. For 10^{25}, $\lambda(q) = 315648\ldots \quad \lambda(p) = 176846\ldots$ if the values are replaced in the inequality (3.17b), we find that we find that $0.556\cdots > m$. In the appendix, you may find List A for $m = 0.5$ and List B for $m = 0.4$, and also list A. For m, if diverse drawings are made according to the values ; and respectively, we may end up with similar curves As in the point $y = 0$ $x = m\lambda(p)$, if we replace this in the equation (3.17). We find that $x = m\lambda(p)$. The points intercepting the axis x are $80, 64, 56, 40, 0.016$ (Figure 3.8)

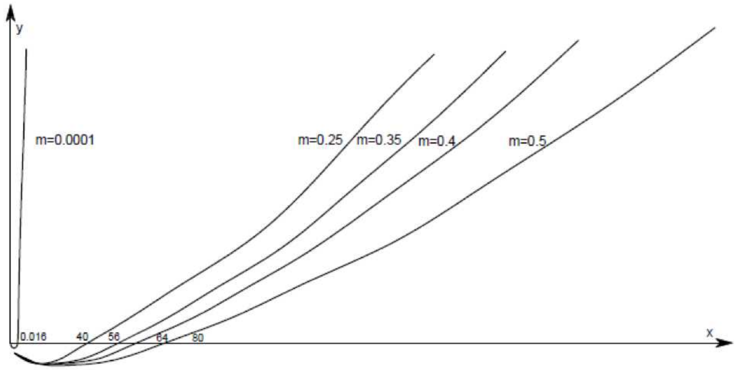


Fig. 3.8

Statistically, we have the available lists $\lambda(p)$ and $\lambda(q)$. In accordance with these values, we had created List A as an example. We may mark these numerical values in an axis x, y. Matching the points we find, we may try to draw a curve. (Figure 3.9) Because of the random distribution of prime numbers, the points that we mark are not placed in an exact order, however they proceed according to the increasing part of the axis x and y. In this case, we will acquire an approximate value of the desired curve or its equation. In order to approximate more to this location, skipping 10 by 10 and drawing a second curve, we acquire a more decent curve compared to the first one (Figure 3.10) There also may be such curves drawn in the units of 100 or 1000. The numbers we see on the curve are the diverse values of n.

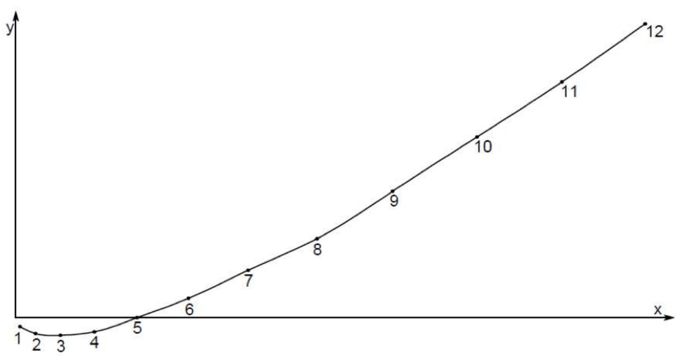


Fig. 3.9

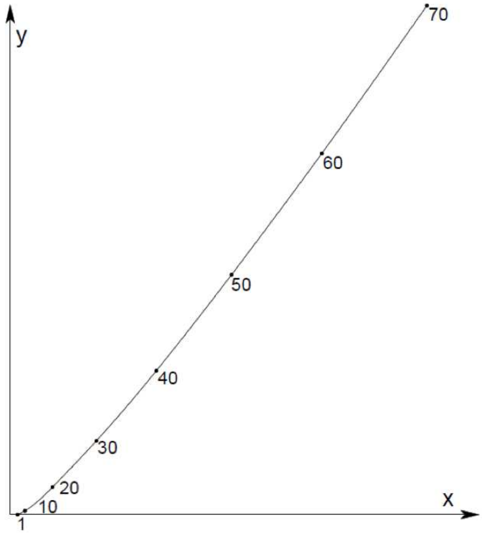


Fig. 3.10

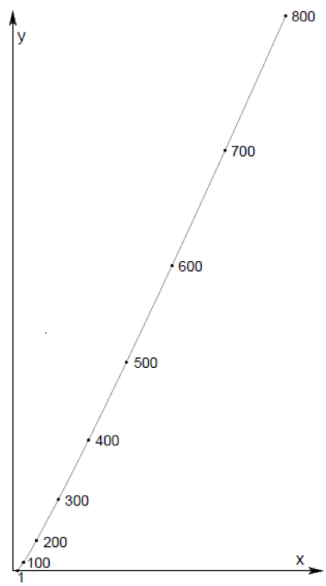


Fig. 3.11

Now, we examine this case on an ellipse.

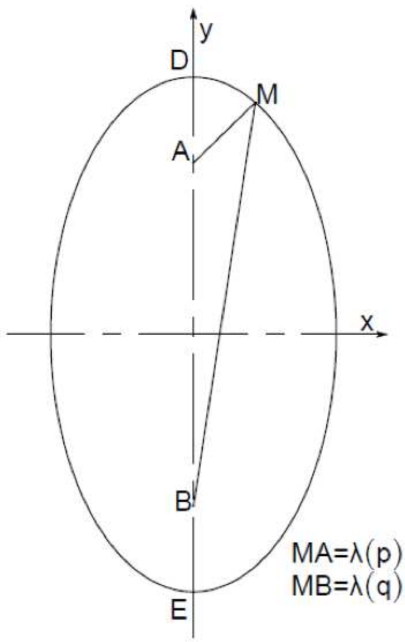


Fig. 3.11a

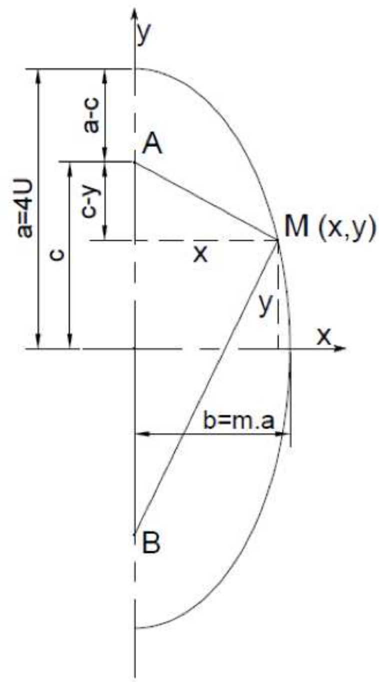


Fig. 3.11b

Fig. 11b' den:

$b = ma = 4U_n m \qquad b > x$

$$4U_n > x \qquad 4U_n > y \tag{3.17c}$$

For $m = \dfrac{1}{2}\ \ 2U_n > x$ and for $m = \dfrac{1}{4}\ \ U_n > x$

In Figure 3.12a, seems to be the final value which $\lambda(p)$ may take.

When the point approximates the point and the point coincides the point :

Assume that $MA = AD$ and $MB = AD + 2c$

$$\frac{BM}{AD} = \frac{AD+2c}{AD} = 1 + \frac{2c}{a-c} = \frac{a+c}{a-c} \text{ and } c = a\sqrt{1-m^2} \tag{3.21}$$

$$\frac{BM}{AD} = \frac{a+a\sqrt{1-m^2}}{a-a\sqrt{1-m^2}} = \frac{1+\sqrt{1-m^2}}{1-\sqrt{1-m^2}} = \frac{(1+\sqrt{1-m^2})^2}{1-1+m^2}$$

$$\frac{BM}{AD} = \left(\frac{1+\sqrt{1-m^2}}{m}\right)^2 \text{ or } \frac{\lambda(q)}{\lambda(p)} = \left(\frac{1+\sqrt{1-m^2}}{m}\right)^2 \tag{3.22}$$

The same limit value for the ratio $\dfrac{\lambda(q)}{\lambda(p)}$

3.2 On Ellipse, $\lambda(\mathbf{p})$ is Perpendicular to Major Axis

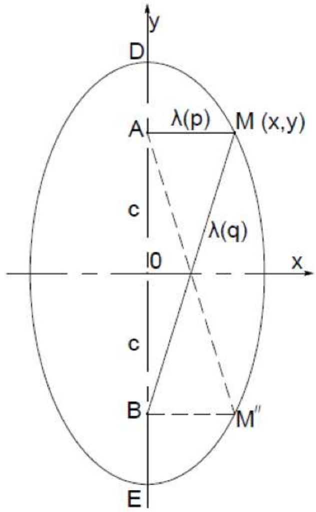


Fig. 12a

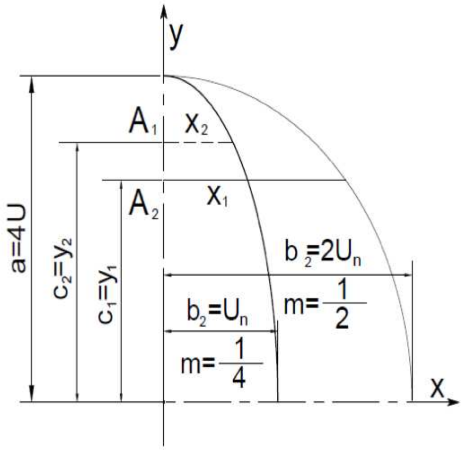


Fig. 12b

$b = ma;$ $\qquad c = a\sqrt{1-m^2}$

From MAB triamgle: $\lambda^2(p) + 4c^2 = \lambda^2(q)$

In the ellipse equation : If we replace as $x = \lambda(p)$ and $y = c$

$x^2 + m^2y^2 = a^2m^2 \qquad \lambda^2(p) + m^2c^2 = a^2m^2$

$\lambda^2(p) = m^2(a^2 - c^2) = m^2b^2 = m^4a^2$

$$\lambda(p) = m^2a \qquad \lambda(p) = 4U_n m^2 \tag{3.23}$$

$\lambda^2(q) = m^4a^2 + 4a(1-m^2) = a^2(m^4 - 4m^2 + 4)$

$\lambda^2(q) = a^2(2-m^2)^2$

$\lambda(q) = a(2-m) \qquad \lambda(q) = 4U_n(2-m^2) \qquad \dfrac{\lambda(q)}{\lambda(p)} = \dfrac{2-m^2}{m^2}$

When $\lambda(p) = x$ and $y = c$, from the equation (3.14) (Fig. 3.13)

$x^2 + m^2y^2 = m^2a^2$ $c = a\sqrt{1-m^2} = 4U_n\sqrt{1-m^2}$

$\left(\lambda(p)\right)^2 + 16U_n^2 m^2(1-m^2) = 16U_n^2 m^2$ the solution leads us again to $\lambda(p) = 4U_n m^2$. For fixed U_n $m = \dfrac{1}{2}$ and according to the values $m = \dfrac{1}{4}$

For $m_1 = \dfrac{1}{2}$ $x_1 = \lambda(p_1) = U_n$ $y_1 = 2\sqrt{3U_n}$

For $m_2 = \dfrac{1}{4}$ $x_2 = \lambda(p_2) = \dfrac{U_n}{4}$ $y_2 = \sqrt{15U_n}$

$x_2 = \dfrac{x_1}{4}$ and $y_2 = \dfrac{\sqrt{5}}{2} y_1$

The case that one of the distance from focal point to ellipse is perpendicular to major axis which is illustrated in figure 3.6, in order to find locus of this case:

We may add the abscissas $\left(\dfrac{\lambda(q)}{\lambda(p)}\right)$ in the points C and D and divide by 2:

$$\left[\frac{2-m^2+\sqrt{3-4m^2}}{m^2} + \frac{2-m^2-\sqrt{3-4m^2}}{m^2}\right] : 2 = \frac{2-m^2}{m^2}$$

When perpendicular case comes up, $r = \dfrac{\lambda(q)}{\lambda(p)}$ value is the middle point of C and D points. (E point) This point is also the midpoint of the two asymptotes. Then

$$r_E = \frac{r_c + r_D}{2} \quad \text{and} \quad 2\frac{\lambda(q)_E}{\lambda(p)_E} = \frac{\lambda(q)_D}{\lambda(p)_D} + \frac{\lambda(q)_C}{\lambda(p)_C} \tag{3.22b}$$

Replacing the the abscisse of the point G $r = \dfrac{2-m^2}{m^2}$ is replaced in the equation (3.21)

The ordinate of the point F: $OE - OA = \dfrac{2-m^2}{m^2} - 1 = \dfrac{2-2m^2}{m^2}$

$FE = AE = \dfrac{2(1-m^2)}{m^2}$

$$\frac{FE}{GE} = 2\sqrt{1-m^2} \quad \text{or} \quad \frac{FE}{GE} = \frac{\lambda(q)-\lambda(p)}{y} = 2\frac{c}{a} \tag{3.23a}$$

For $m = \dfrac{1}{2}$ $\lambda(p) = 4U_n m^2$ (3.23) $\lambda(p) = U_n$ is found.

Tam değeri arayalım: $m_1 = \dfrac{1}{2}$ $\lambda(q) = 7\lambda(p)$

$\lambda(q) + 7\lambda(p) = 8U_n$ $8\lambda(p) = 8U_n$

In Table A, U_n is divided by 8 that we found is equal to $\lambda(p)$ $n = 45756$

$U_n = 3140440182$ $\quad \lambda(p) = 31404339398$ $\quad x = 3140439397.999987\ldots$ There is a difference like $U_n - \lambda(p) = 784$. This may come from that $\lambda(p)$ is not perpendicular. (Like c is not perpendicular one of the m values). As we have discussed on the page 57, we find the desired value.

$$\lambda(q) = a(2-m^2)$$

$$\frac{\lambda(p)}{\lambda(q)} = \frac{am^2}{a(2-m^2)} = \frac{m^2}{2-m^2} \quad \frac{\lambda(q)}{\lambda(p)} = \frac{2-m^2}{m^2} \quad m = \sqrt{\frac{2}{1+r}} \tag{3.23b}$$

$$m_1 = \frac{1}{2} \quad \frac{\lambda(q_1)}{\lambda(p_1)} = 7 \qquad m_2 = \frac{1}{4} \quad \frac{\lambda(q_2)}{\lambda(p_2)} = 31$$

3.3 $\lambda(p)$ and $\lambda(q)$ are Perpendicular: $[\lambda(\mathbf{p}) \perp \lambda(\mathbf{q})]$

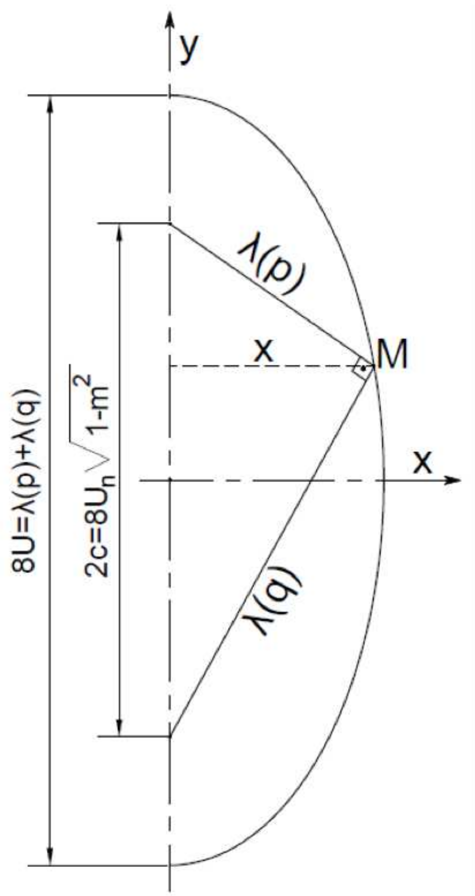


Fig. 3.12c

$$\lambda^2(p)+\lambda^2(q)=4c^2 \qquad c=4U_n\sqrt{1-m^2} \qquad \lambda(p)\lambda(q)=x.2c$$

$$x=\frac{\lambda(p)\lambda(q)}{8U_n\sqrt{1-m^2}}; \qquad y=\frac{\lambda(p)-\lambda(q)}{2\sqrt{1-m^2}}$$

$$\frac{y}{x}=\frac{\lambda(q)-\lambda(p)}{2\lambda(p)\lambda(q)}(\lambda(q)+\lambda(p))=\frac{1}{2}\left(\frac{\lambda(q)}{\lambda(p)}-\frac{\lambda(q)}{\lambda(p)}\right)$$

$$t=\frac{y}{x}; \qquad r=\frac{\lambda(q)}{\lambda(p)}; \qquad t=\frac{1}{2}=\left(\frac{r^2-1}{r}\right);$$

$$t=\frac{r-1}{m\sqrt{4r-m^2(r+1)^2)}} \tag{3.21}$$

Solving these equations::

$$m = \frac{\sqrt{2r}}{r+1}; \qquad r = \frac{1-m^2+\sqrt{1-2m^2}}{m^2};$$
$$t = \frac{\sqrt{1-2m^2}}{m^2}; \qquad t = r - \frac{1-m^2}{m^2}$$

Using this last relation, we may show the place of the point M in Figure 3.6, in

Figure 3.6a. (the point H)

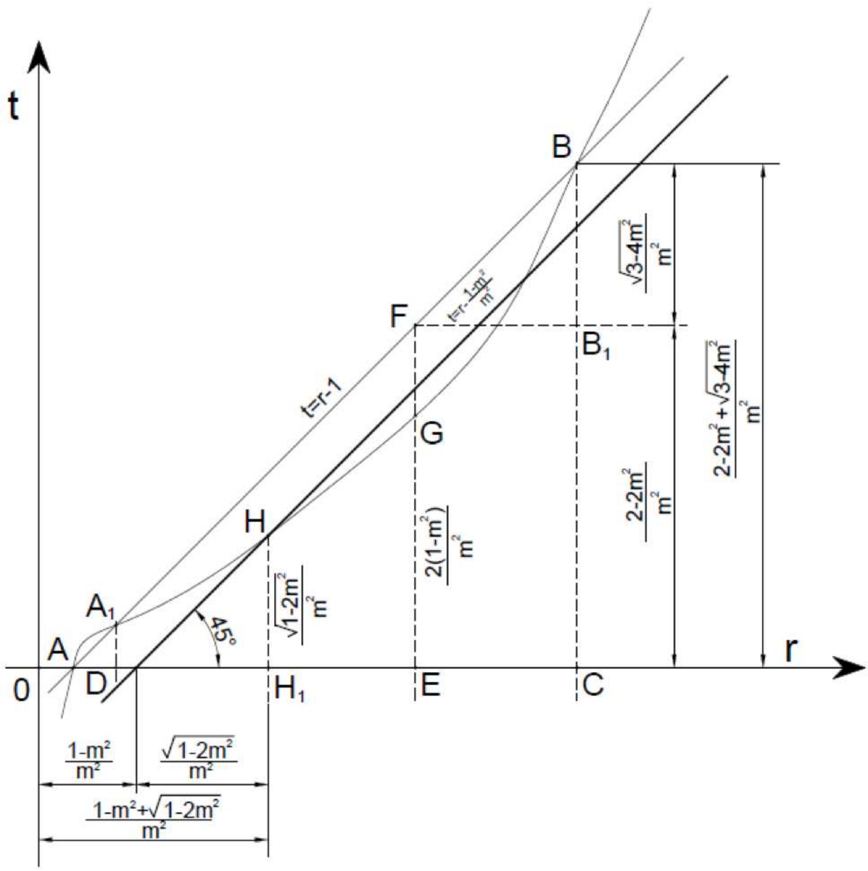


Fig. 3.6a

We may move the orthogonal positions shown in Figure 3.12a and 3.12 to a single drawing.

1.) $\lambda(p) \perp y$ axis

2.) $\lambda(p) \perp \lambda(q)$

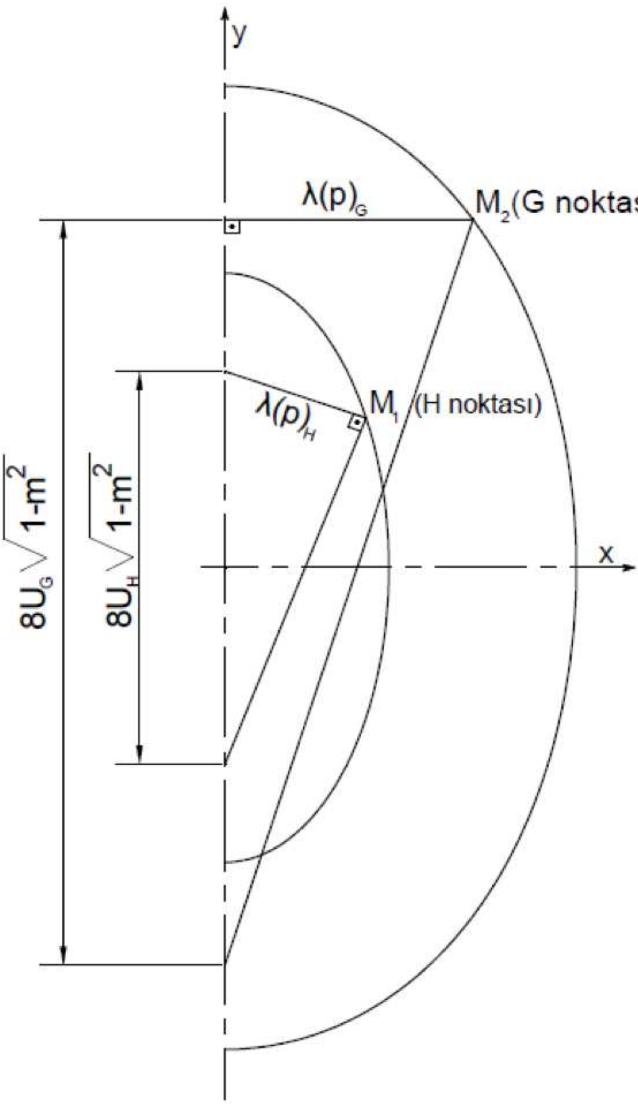


Fig. 3.12d

$\lambda(p)_G = 4U_G m^2$

$\lambda^2(p)_H + \lambda^2(q)_H = 64U_H^2(1-m^2)$ If m^2 is removed between these two equations:

$$\left(\frac{\lambda(p)_G}{8U_G}\right)^2 + \left(\frac{\lambda(p)_H}{8U_H}\right)^2 = \left(1 - \frac{\lambda(p)_H}{8U_H}\right)^2 \tag{3.22c}$$

We have found another relation between the points H and G independent from (m).

Checking: $\left(m = \dfrac{1}{2}\right)$

$\dfrac{\lambda(q)_G}{\lambda(p)_G} = 7 \qquad n \cong 46000$

$\lambda(p)_G = 3172614519; \quad \lambda(q)_G = 22219569481$

$\dfrac{\lambda(q)_H}{\lambda(p)_H} = 5.828\ldots \qquad n \cong 7900$

$\lambda(p)_H = 109716964; \quad \lambda(q)_H = 639234636$

$8U_G = 25392184000 \quad 8U_H = 748951600$

$0.037071\ldots \cong 0.037249\ldots$ is found.

This difference stems from the fact that the exact corresponding n's for 7 and 5.828427... are not found in our list.

3.4 Limit Values

The limit values shown in Figure 3.6:

$$\frac{\lambda(q)}{\lambda(p)} = \left(\frac{1+\sqrt{1-m^2}}{m}\right)^2 \tag{3.22}$$

$$m_2 = \frac{1}{2} \quad \frac{\lambda(q_1)}{\lambda(p_1)} \cong 13.928\ldots \qquad m_1 = \frac{1}{4} \quad \frac{\lambda(q_2)}{\lambda(p_2)} \cong 61.983\ldots$$

So it never happens twice selected for $\frac{\lambda(q)}{\lambda(p)}$ values in the case that the m value perpendicular to the major axis. x is undefined if it happens
$2\frac{2-m^2}{m^2} > \left(\frac{1+\sqrt{1-m^2}}{m}\right)^2$ If this inquality is solved, we find that $m > 0$.
Limit values r_L; since the case of perpendicular r_E, we may find the value $2r_E - r_L = ?$ value. (3.22a) formula:

$$2r_L r_E - r_L^2 = 1 \text{ ve } r_E = \frac{1+r_L^2}{2r_L} \qquad 2.\frac{1+r_L^2}{2r_L} - 1 = \frac{1}{r_L} = \left(\frac{m}{1+\sqrt{1-m^2}}\right)^2$$

The difference between them is the inverse of the limit value. In this case, if $ON = 2OE$ in Figure 3.6, then $OL.LN = 1$.
As $m \neq 1$ and $m \neq 0$, $\left(\frac{1+\sqrt{1-m^2}}{m}\right)^2$ is always a finite value. When x and y are infinite, we find that $\frac{y}{x} = \frac{\infty}{\infty}$. However, we do not know if the limit is infinite. The equation of the curve shown in Figure 3.6:

$$t = \frac{r-1}{m\sqrt{4r-m^2(r+1)^2}} \qquad (3.21) \qquad t = \frac{y}{x}; \qquad r = \frac{\lambda(q)}{\lambda(p)}$$

For every m, the value of r changes. In general sense, the limit values found in a function are absolutely valid. However, in practical sense, we cannot put a limit for $\frac{\lambda(q)}{\lambda(p)}$. (Because, for example, for $m = \frac{1}{2}$, $\frac{\lambda(q)}{\lambda(p)} = 13.928...$)
In the prime number list that I could access, for 10^{25}, $\frac{\lambda(q)}{\lambda(p)} = 17.848....$
If $\frac{\lambda(q)}{\lambda(p)} = 13.928\ldots$, , it must be somewhere between $10^{19} \sim 10^{20}$ (Table 12)

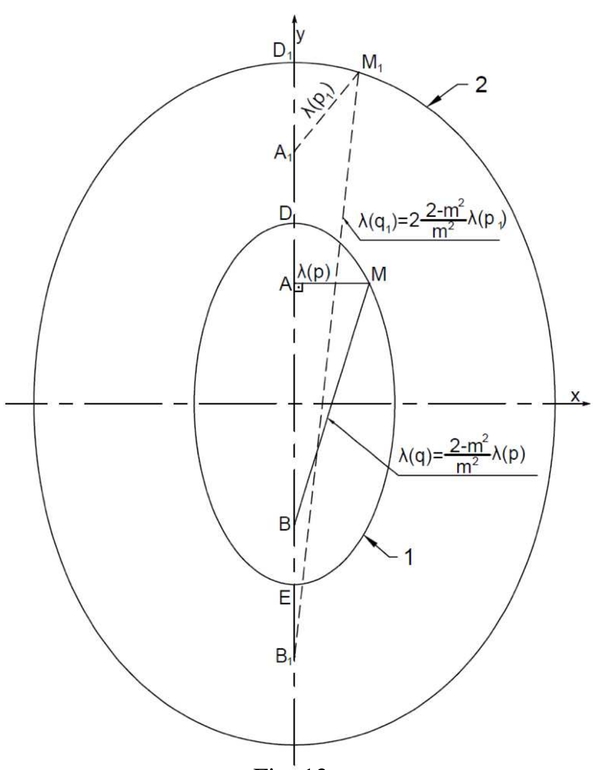


Fig. 13

The ellipse no. 1 is the one where $\lambda(p)$ is perpendicular to major axis is located and the major axis is .

The ellipse no. 2 is the one where twice of the BM is located and the major axis is $2a_1$. If this ellipse exists, $A_1M_1 > A_1D_1$.

$A_1D_1 = a_1 - c_1; \quad c_1 = a_1\sqrt{1-m^2}; \quad A_1D_1 = a_1(1-\sqrt{1-m^2})$

$\lambda(q_1)+\lambda(p_1) = 2a_1$ and $2\frac{2-m^2}{m^2}\lambda(q_1)+\lambda(p_1) = 2a_1$

$a_1 = \frac{4-m^2}{2m^2}\lambda(p_1) \qquad A_1M_1 = \lambda(p_1)$

$\lambda(p_1) > a_1(1-\sqrt{1-m^2}) = \frac{4-m^2}{2m^2}(1-\sqrt{1-m^2})\lambda(p_1);$

$2m^2 > (4-m^2)(1-\sqrt{1-m^2})$ We may find that $-m^6 > 0$ Then, $A_1D_1 > A_1M_1$

Therefore, on this ellipse, there is no point providing that $\lambda(q_1) = 2\frac{2-m^2}{m^2}\lambda(p_1)$ The minimum value of A_1M_1 is A_1D_1. Before M_1 reaches the point D_1, or when it reaches that point, $\lambda(p_1)$ becomes infinite. We may rewrite the equation (3.17)and we may examine the case where the radicand inside the square root is equal to zero. $x = \frac{m}{2\sqrt{1-m^2}}\sqrt{4\lambda(p)\lambda(q) - m^2[\lambda(p)+\lambda(q)]^2}$ and $4\lambda(p)\lambda(q) = m^2[\lambda(p)+\lambda(q)]^2$ In this case, $x = 0$. But is it possible? We assumed that the locus curve is progressing as it increases. If , this curve needs to rotate and intersects the axis at some point. This may apply to finite ellipsis. What can be the answer for an infinite ellipse? What is the distance between the most approximate point to and ? If the ellipse is infinite, is the distance between this point and D infinite?

We may return to the equation (3.19):

$$\frac{[\lambda(q)+\lambda(p)]^2}{4(1-m^2)} = t^2m^2\left[\frac{[\lambda(q)+\lambda(1)]^2}{4} - \frac{[\lambda(q)-\lambda(p)]^2}{4(1-m^2)}\right] \tag{3.19}$$

When $\lambda(q) = \lambda(p)$, there is only one point.From this point (from $n > 5$) the infinite, for all of the points will be $\lambda(q) \neq \lambda(p)$ and therefore, $t \neq 0$. Then $\frac{[\lambda(q)+\lambda(p)]^2}{4} - \frac{[\lambda(q)-\lambda(p)]^2}{4(1-m^2)} \neq 0$. If 0 ,the left side of the equation (3.19) is also zero. However, we accepted that $\lambda(q) \neq \lambda(p)$.

$$\begin{aligned}
&\frac{[\lambda(p)+\lambda(q)]^2}{4} - \frac{[\lambda(q)-\lambda(p)]^2}{4(1-m^2)} \\
&= \frac{[\lambda(p)+\lambda(q)]^2(1-m^2) - [\lambda(q)-\lambda(p)]^2}{4(1-m^2)} \\
&= \frac{4\lambda(p)\lambda(q) - m^2[\lambda(p)+\lambda(q)]^2}{4(1-m^2)}
\end{aligned}$$

In this way, we have found the value of x inside the square root in the equation (3.17). We also have seen that this value is not equal to 0. Then, x cannot be 0 $x \neq 0$.

As the point M approaches the point D, it exceeds the limit $\dfrac{\lambda(q)}{\lambda(p)}$

We may examine the state of the limit: $\dfrac{\lambda(q)}{\lambda(p)} = \left(\dfrac{1+\sqrt{1-m^2}}{m}\right)^2$

$AD = a - c = a(1-\sqrt{1-m^2})$ $\qquad AD = a(1-\sqrt{1-m^2})$

$MA = \lambda(p)$ ve $\lambda(p) + \lambda(q) = 2a$:

$\lambda(q) = \lambda(p)\left(\dfrac{1+\sqrt{1-m^2}}{m}\right)^2 \quad \lambda(p) + \lambda(q)\left(\dfrac{1+\sqrt{1-m^2}}{m}\right)^2 = 2a$

$\lambda(p) = a\dfrac{2m^2}{2+2\sqrt{1-m^2}} = a\dfrac{m^2(1-\sqrt{1-m^2})}{1-1+m^2} = a(1-\sqrt{1-m^2})$

$MA =$ From $\lambda(p) = a(1-\sqrt{1-m^2})$ $AD = MA$ is found.

For an infinite ellipse, if the distance between the most approximate point to D and D is infinite, the locus curve can become infinite without rotating to the axis y, as it increases. Surely, this is an assumption. Now, using another method, let us examine this assumption and whether $x = 0$ or not.

$$x = \frac{m}{2\sqrt{1-m^2}}\sqrt{4\lambda(p)\lambda(q) - m^2(\lambda(p)+\lambda(q))^2}$$

$$\frac{4(1-m^2)}{m^2}x^2 = 4\lambda(p)\lambda(q) - m^2(\lambda(p)+\lambda(q))^2$$

$$\frac{(1-m^2)}{m^2}x^2 = \lambda(p)\lambda(q) - m^2\left(\frac{\lambda(p)+\lambda(q)}{2}\right)^2 \quad x^2 + m^2y^2 = m^2a^2$$

$x^2 + m^2y^2 = m^2\left(\dfrac{\lambda(p)+\lambda(q)}{2}\right)^2$ We may add up.

Now that we end up with two ellipse equations,
$\dfrac{x^2}{m^2} + m^2y^2 = \lambda(p)\lambda(q)$ and $x^2 + m^2y^2 = m^2\left(\dfrac{\lambda(p)+\lambda(q)}{2}\right)^2$ we may regard these equations as a Pythagorean relation and draw the right triangles with using the hipotenus values of:

$m\left(\dfrac{\lambda(p)+\lambda(q)}{2}\right)$ and $\sqrt{\lambda(p)\lambda(q)}$

$AB = \dfrac{x}{m}$; $AE = x$ and $AC = my$ (Fig. 3.14)

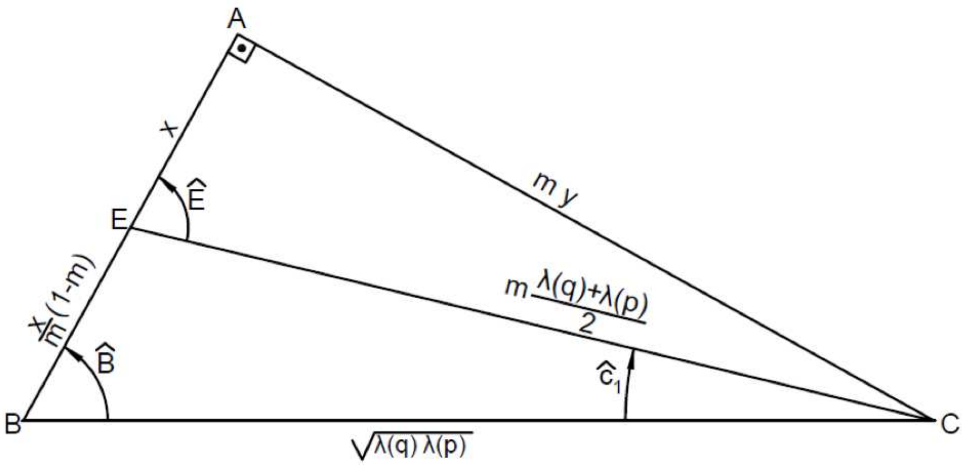


Fig. 3.14

$\hat{E} = \hat{B} + \hat{C}_1$,therefore $\hat{E} > \hat{B}$. In the triangle ABC, $\tan\hat{B} = \frac{my}{\frac{x}{m}} = m^2\frac{y}{x}$;

In the triangle AEC, $\tan\hat{E} = m\frac{y}{x}$; $\quad \frac{\tan\hat{B}}{\tan\hat{E}} = m$. In list A, for $m = \frac{1}{2}$ and $n = 38$, there is an approximate value of $y \cong x$. After this value, being $y > x$ it always increases. As y progresses to infinity, the angles B and E are also increasing in the arrow direction, approximating to 90° in the infinite degree. In this position :

$(\sin 90 = 1)$ and $(\cos 90 = 0)$

$\sin B = \frac{my}{\sqrt{\lambda(p)\lambda(q)}} = 1 \quad y$ and $\lambda(p)$, $\lambda(q)$ goes to infinity, we may end up with a ambiguous case of $\frac{\infty}{\infty}$ however the limit is 1.

As $\cos B = \frac{\frac{x}{m}}{\sqrt{\lambda(p)\lambda(q)}} = 0 \quad x$ and $\lambda(p)$, $\lambda(q)$ goes to infinity,the limit of this indeterminate case is 0. We may examine the case of exceeding to infinity, in a right triangle we draw, assuming that the values$y$and $\sqrt{\lambda(p)\lambda(q)}$.

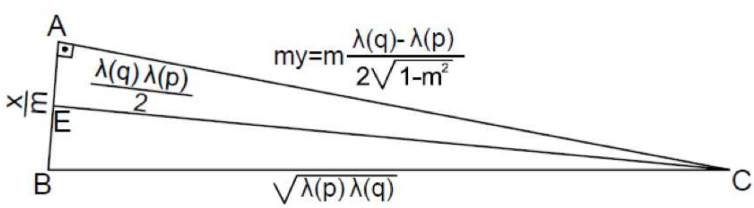


Fig. 3.15

$\hat{B} = 90°$ approximates to $\hat{C}$ lessens and the triangles approximastes isosceles triangles and $AC = EC = BC$.

$$m\frac{\lambda(q) - \lambda(p)}{2\sqrt{1-m^2}} = m\frac{\lambda(p) + \lambda(q)}{2} = \sqrt{\lambda(p)\lambda(q)}$$

In any case,we may find the equation providing the limit values of $r = \dfrac{\lambda(q)}{\lambda(p)}$ that we have found beforehand.

$$m^2r^2 - 2r(2-m^2) + m^2 = 0 \text{ or } \frac{\lambda(q)}{\lambda(p)} = \left(\frac{1 \pm \sqrt{1-m^2}}{m}\right)^2$$

We have found the same limit values before. The angle $\hat{B}$ according to 90°, and the angle $\hat{C}$ according to ∞, goes to zero, while narrowing. However, the arms of the angle, my and $\sqrt{\lambda(p)\lambda(q)}$ goes to infinity, while expanding. If the distance between these two ends is $\dfrac{x}{m}$, it will eventually be infinite. Therefore, there is no case of $x = 0$.

3.5 Finding a Composite Number out of the Prime Number

We have used the following method for the calculation of U_n's generating the prime numbers such as the 100000000 and 1000000000th prime numbers. If we examine 2 and 3 prime numbers , there are known $\lambda(p) = 100000000 - 2 = 99999998$ as amount of primes and p is prime number. In order to find the values $8U_n$ and $\lambda(q)$ we may use the formula $Q_n = \sqrt{24U_n + 1}$

1. p: The known prime number
2. $\lambda(p)$: The quantity of the prime numbers
3. Adding 1 or 2 to $\sqrt{p}$, we search for the most approximate value of Q_n^2 that is greater than $\sqrt{p}$ and not divided by 3.(Even though ($Q_n^2 > p$), sometimes the most approximate value to Q_n^2 may not be p.)

4 The prime number (p) on the list of prime numbers and lesser than and closer to this value is found. Its ranking on the list is also found. $[\lambda(p)]$

5. From $U_n = \dfrac{Q^2 - 1}{24}$, we may calculate U_n and $8U_n$
6. $\lambda(q) = 8U_n - \lambda(p)$

The values of $\lambda(q)$ and $\lambda(p)$ are found and if we put these values to inequality (3.17a), it provides maximum m value.

Example 1

100000000th prime number $p = 2038074743$

$\sqrt{p} = 45145.04 < 45145 + 2 = 45147$ As this number is divided by 3, it cannot be a number in the sequence of Q_n Therefore: Although $Q_n = 45149$

$Q_n^2 = 2038432201 > 2038074743$, p is not the most approximate prime number that we are looking for. Out of the prime number lists, the prime number lesser than and most approximate to Q_n^2 is found 2038432177 This number is the 100016627th number in the prime number list. Regarding the absence of 2 and 3

$\lambda(p) = 100016625 \qquad U_n = \dfrac{(45149)^2 - 1}{24} = 84934675$

$8U_n = 679477400 \qquad \lambda(p) = 100016625 \qquad \lambda(q) = 579460775$

Example 2

The quantity of prime numbers	: 500000000
The prime number	: 11037271757
$\sqrt{11037271757}$	$= 105058.4 + 1$
Q_n	$= 105059$
Q_n^2	$= 11037393481$
The most approximate prime number	: 11037393427
Ranking	: 500005356
$U_n = \dfrac{(105059)^2 - 1}{24}$	$= 459891395$
$8U_n$	$= 3679131160$
$\lambda(p)$	$= 500005354$
$\lambda(q)$	$= 3179125806$

Example 3:

The quantity of prime numbers	: 1000000000
The prime number	: 22801763489
$\sqrt{22801763489}$	$= 151002.5 + 1$
Q_n	$= 151003$
Q_n^2	$= 22801906009$
The most approximate prime number	: 22801906007
Ranking	: 1000005995
$U_n = \dfrac{(151003)^2 - 1}{24}$	$= 459891395$
$8U_n$	$= 7600635336$
$\lambda(p)$	$= 1000005993$
$\lambda(q)$	$= 6600629343$

Example 4:

The quantity of prime numbers	: 1400000000
The prime number	: 32416190071
$\sqrt{:32416190071}$	$= 180044.96 - 1$
Q_n	$= 180043$<u>(*)</u>
Q_n^2	$= 32415481849$
The most approximate prime number	: 32415481843
Ranking	: 1399970567
$U_n = \dfrac{(180043)^2 - 1}{24}$	$= 459891395$
$8U_n$	$= 10805160616$
$\lambda(p)$	$= 1399970565$
$\lambda(q)$	$= 9405190051$

<u>(*)</u>

$Q_n = 180043$ (At the beginning, as we could not find a list that generates a greater ranking than the $1400000000th$, we chose the one lesser than the 180044.96)

3.6 Finding Sum Q_n, According to The Number 10^n

	$\lambda(p)+\lambda(q)$
10	2
10^2	30
10^3	330
10^4	3330
10^5	33330
10^6	333330

....

Regarding the sorting above, we may write the formula below. We may assume that $\lambda(10^n) = \lambda(p) + \lambda(q)$.

$\cancel{\lambda(10^{n-3})}$ $=310^{n-4}+\cancel{\lambda(10^{n-4})}$

$\cancel{\lambda(10^{n-2})}$ $=310^{n-3}+\cancel{\lambda(10^{n-3})}$

$\cancel{\lambda(10^{n-1})}$ $=310^{n-2}+\cancel{\lambda(10^{n-2})}$

$\lambda(10^n)$ $=310^{n-1}+\cancel{\lambda(10^{n-1})}$

....

$$\lambda(10^n) = 3\sum_{2}^{n} 10^{n-1} \tag{3.24}$$

Given that $n = 1$, $\lambda(10^1) = 3$. Here, Q_n sequence contains 5 and 7 as a less then 10 , also 1 is not consired as prime number by definition of prime number that leads it may be composite number. In order not to have a contradiction, being $n > 2$ this formula is valid.

Chapter 4

NUMERICAL MODELING

4.1 Find the Locus Curve [$y = G(x)$]

The equations that are available and will be used for the calculation of $\lambda(p)$:

1.

$$x^2 + m^2 y^2 = 16U_n^2 m^2 \tag{3.14}$$

2.

$y = G(x)$ locus curve

3.

$$\lambda(p) = \sqrt{(y-c)^2 + x^2} \tag{3.12a}$$

Note: $c = 4U_n\sqrt{1-m^2}$

$$\lambda(p) = 4U_n - y\sqrt{1-m^2} \tag{3.14d}$$

Replacing the values x and y that we will find from the solutions of the equations 1 and 2 in (3.12a) or (3.14d), If we find a function of $x = G(y)$, we may find $\lambda(p)$ without a second operation.

I.

From the studies I have made by the help of tables and lists, I have seen that there is a relation between n, x,y. However, these are distant values for $\lambda(p)$.

$m = 0.50$ Table 13

n	$n = y^{\frac{lnx-1}{lnx+lny}}$	$n = \sqrt[4]{(xy)^{1-\frac{1}{lnx}}}$	$n = \sqrt[4]{e^{lnxy-2}}$	$n = e^{\frac{lnx(lny-1)}{lnxy}}$
40	39.772	39.772	40.995	40.027
200	206.375	206.828	209.025	210.647
1000	1031.793	1036.462	1050.121	1058.451
5000	5094.781	5126.907	5198.885	5234.878
15000	15093.587	15203.877	15420.344	15514.373
25000	24988.079	25181.369	25540.944	25686.711
40000	39717.867	40040.1734	40612.803	40829.996
45000	44605.850	44971.991	45615.322	45855.167
50000	49484.833	49895.136	50609.035	50871.070
55000	54355.612	54810.371	55594.720	55878.531
60000	59218.930	59718.405	60573.094	60878.305
69000	67955.688	68536.252	69517.307	69860.192

In the table I have made only for the value of $m = 0.50$, considering that the exponential functions are intersecting the origin, it is assumed that the (x) value is numerically $(x-80)$. In the formula $n = \sqrt[4]{(xy)}1 - \dfrac{1}{lnx}$ for $n = 43000$ the margin of error is 0. In the formula $n = y^{\frac{lnx-1}{lnx+lny}}$ at $n \cong 250000$, error margin is 0.

II.

In the function $y = x^k$ $k = \frac{\log y}{\log x}$, $A_1 = \frac{2k}{k^2+1}$ and $A_2 = \frac{k}{e^{k-1}}$.

$m = 0.50$

n	A_1	A_2
1000	0.998700473	0.998677555
5000	0.998529401	0.998501721
25000	0.998480406	0.988451309
69000	0.998473211	0.998443908

Here,$A_1 \sim A_2$, $2e^{k-1} \sim (k^2+1)$ $1.1 > k > 1$, this formula generates better values. For a better approximate value, I have tried to find a solution as following.

$$2e^{k-1} = (k^2+1)$$
$$k-1 = \log(k^2+1) - \log 2$$
$$\Delta_k = k - \log(k^2+1) - (1-\log 2) \text{ where } \Delta_k \text{ is difference of k}$$
$$1-\Delta_k = 1 - [k - \log(k^2+1) - (1-\log 2)]$$
$$1-\Delta_k = 2 - k + \log(k^2+1) - \log 2$$

$m = 0.50$

n	$1-\Delta_k$	$\frac{A_1}{A_2}$
1000	0.999977031	0.999977052
5000	0.999972278	0.999972279
25000	0.999970861	0.999970861
69000	0.999970651	0.999970651

$$1-\Delta_k = 2 - k + \log(k^2+1) - \log 2$$

Since Δ_k is so small $\dfrac{k^2+1}{2e^{k-1}} \cong 1$

$$\log e^2 - \log 2 + \log(k^2+1) - e^k = \frac{k^2+1}{2e^{k-1}}$$
$$\log\frac{e^2}{2} + \log\left(\frac{k^2+1}{e^k}\right) = \log\left[\frac{e^2}{2}\frac{k^2+1}{e^k}\right] = \frac{k^2+1}{2e^{k-1}}$$
$$\log\left[e.\frac{k^2+1}{2e^{k-1}}\right] = \frac{k^2+1}{2e^{k-1}}$$
$$1+\log\left[e.\frac{k^2+1}{2e^{k-1}}\right] = \frac{k^2+1}{2e^{k-1}}$$
$$\frac{k^2+1}{2e^{k-1}} - \log\left[\frac{k^2+1}{2e^{k-1}}\right] = 1 \qquad k = \frac{\log y}{\log x}$$
$$\frac{\left(\frac{\log y}{\log x}\right)^2+1}{2e^{\frac{\log y}{\log x}-1}} - \log\frac{\left(\frac{\log y}{\log x}\right)^2+1}{2e^{\frac{\log y}{\log x}-1}} = 1 \tag{4.25}$$

We have found an equation that generates better approximate values (average 1 :10^9). Our first equation $x^2 + m^2y^2 = 16U_n^2m^2$ (3.14) $m = \dfrac{1}{2}$ Replacing these: $4x^2 + y^2 = 16U_n^2$. Despite all my efforts, I could not find the common solution that generates x and y out of these two equations. If a mathematician achieves this, he or she may find the approximate value of $\lambda'(p)$ by calculating the values x and y. Perhaps, the fact that the equation (4.25) is a general equation that is valid for the lesser k numbers is the reason why it does not provide a solution.

III.

In order to find a function $[G(x)]$ by the use of the coordinates located in the A lists, I have contacted with Prof. Taylan AKDOĞAN from Boğaziçi University. He was interested in the subject and using the List as a base, he has done necessary researches and sent me the empirical formula of (4.26). Using this formula, we have started to search for a common solution, as I have stated at the beginning of the chapter 3. By use of the coordinates$x$,y given in the list A , it is empirically modelled with the 8-parameter function of

$$y(x) = C + e^{w} \sum_{k=0}^{6} a_k w^k \qquad w \equiv \log(x+22) \tag{4.26}$$

The errors found in the data are found by the analyses of the local deviations. Then, by the minimization of χ^2, the fit parameters are found. These fit values, with the least value of $\dfrac{\chi^2}{\text{d.o.f.}} = 102$

Parameter	Fit Value $\pm$ Error (σ_{a_k})
a_0	$-1.029755385051804 \times 10^{0} \pm 1.092521628799510 \times 10{-}3$
a_1	$3.032868938369940 \times 10^{-1} \pm 3.206790551525189 \times 10^{-4}$
a_2	$-6.625190802400414 \times 10^{-3} \pm 4.014581191741933 \times 10^{-5}$
a_3	$-1.986462693223385 \times 10^{-4} \pm 2.743346520478255 \times 10^{-6}$
a_4	$2.816419279298372 \times 10^{-5} \pm 1.075251334079647 \times 10^{-7}$
a_5	$-9.149075456434692 \times 10^{-7} \pm 2.278894541808835 \times 10^{-9}$
a_6	$1.135567018200240 \times 10^{-8} \pm 2.028433124194171 \times 10^{-11}$
C	$-5.835147466499460 \times 10^{1} \pm 1.167814389288842 \times 10^{0}$

At the same time,we also had another formula.In (3.14),replacing $m = \dfrac{1}{2}$

$$4x^2 + y^2 = 16U_n^2$$

With the common solution of these equations, we may find the approximate value of x. Firstly, we may rewrite the ellipse equation.

$y = \sqrt{16U_n^2 - 4x^2}$

Now we may equalize the both sides.

$$\sqrt{16U_n^2 - 4x^2} = C + e^w \sum_{k=0}^{6} a_k w^k$$

We may add C values to left side of equation.

$$\sqrt{16U_n^2 - 4x^2} - C = e^w \sum_{k=0}^{6} a_k w^k$$

We may divide e^w to both side.

$$\frac{\sqrt{16U_n^2 - 4x^2} - C}{e^w} = \sum_{k=0}^{6} a_k w^k$$

We may expand sum sign.

$$\frac{\sqrt{16U_n^2 - 4x^2} - C}{e^w} = a_0 + a_1 w + a_2 w^2 + a_3 w^3 + a_4 w^4 + a_5 w^5 + a_6 w^6$$

C and a_0 are constant in order to easer computation we may add a_0 the other side.

$$\frac{\sqrt{16U_n^2 - 4x^2} - C}{e^w} - a_0 = a_1 w + a_2 w^2 + a_3 w^3 + a_4 w^4 + a_5 w^5 + a_6 w^6$$

We may replace $w = \log(x + 22)$

$$\frac{\sqrt{16U_n^2 4x^2} - C}{e^{log(x+22)}} - a_0 = a_1 log(x+22) + a_2 log^2(x+22) +$$
$$a_3 log^3(x+22) + a_4 log^4(x+22) + a_5 log^5(x+22) + a_6 log^6(x+22)$$

The right side of the equation:

$$f(x) = \frac{\sqrt{16U_n^2 - 4x^2} - C}{e^{log(x+22)}} - a_0$$

The left side of the equation:

$$g(x) = a_1 log(x+22) + a_2 log^2(x+22) + a_3 log^3(x+22) + a_4 log^4(x+22) + a_5 log^5(x+22) + a_6 log^6(x+22)$$

$\forall \varepsilon > 0 \qquad |f(x) - g(x)| \leq \varepsilon \qquad \varepsilon \longrightarrow 0$ as $x \longrightarrow x_0$.

Assuming the various x values, the common solution of the equation approximates.

In this way, when we replace the values of x_0 and $y_0 = \sqrt{16U_n^2 - x_0^2}$ in the formula (3.12a)

$\lambda'(p) = \sqrt{(c-y)^2 + x^2} \quad \Delta\lambda(p) = \lambda(p) - \lambda'(p)$

Note: $\lambda(p)$: real quantity of prime numbers

$\lambda'(p)$: the quantity of prime number that we have found.

Table 14 is found by this analysis. If paid attention, from 10^{12}, $\Delta\lambda(p)$ generates really great values. The reason for that is the fact that the coordinates used in the formula (3.24) are up to $n = 30000$. The study we have conducted for $m = \frac{1}{2}$ is only up to the limit value of $\frac{\lambda(p)}{\lambda(q)} = 13.928$. In Table 12 this limit value ends around 10^{20} (For 10^{20} $\frac{\lambda(p)}{\lambda(q)} = 14{,}0094 > 13.928...$)

Table 14

	$\pi(x)$	Δ Riem.	$\Delta\lambda'(p)$
10	4		
10^2	25	1	
10^3	168	0	
10^4	1229	-2	0
10^5	9592	-5	24
10^6	78498	29	18
10^7	664579	88	131
10^8	5761455	97	137
10^9	50847534	-79	138
10^{10}	455052511	-1828	1736
10^{11}	4118054812	-2318	2072
10^{12}	37607912018	-1476	357339
10^{13}	346065536839	-5773	21035410
10^{14}	3204941750802	-19200	218795031
10^{15}	29844570422669	73218	13691417801
10^{16}	279238341033925	327052	745220710841

4.2 Studying the Optimum m Values

For a common solution, estimating the value of (x) is more difficult than estimating the value of, (y). On page 57, in the equation of (3.17c) is not dependant on $(4U_n > y)m$. As (y) approaches to the greater values of 10^n , it approximates to $4u_n$ and to the limit value.

$m = 0.50$ Table 15

10^n	x	y	$4U_n$
10^{12}	37314747673.8180176900160371309	149024213473.446630119072065699	166666666665
10^{14}	3006398488941.26604674401087399	15544261007860.3905918587558374	16666666666665
10^{19}	101595256121524259.88739730006125	1654234382882965546.5213458665233	1666666666666666665

The coordinates calculated for $m = 0.5$ will end around 10^{20} as we acknowledge. On the lists of the greatest prime numbers, $\dfrac{\lambda(q)}{\lambda(p)} = 17.848...$. To exceed this limit, I have made operations according to the value m. The limit values are easily found on the formula (3.21d).

$$r = \frac{\lambda(q)}{\lambda(p)} = \left(\frac{1+\sqrt{1-m^2}}{m}\right)^2 \tag{3.21d}$$

In order to find that up to which n this value is, I have found some practical results for 10^n, making use of the Table 12.

Table 16

n	r	n	r	n	r
2	0.3043478260869565	10	6.325161962792298	18	12.47348218418118
3	0.9879518072289157	11	7.094436538571851	19	13.24150454938001
4	1.713936430317848	12	7.863383140978044	20	14.00947366228904
5	2.475495307612096	13	8.632086927222776	21	14.7773977159671
6	3.246458418263351	14	9.400605042200693	22	15.54528329690413
7	4.015716764197377	15	10.16897742579544	23	16.31313576620591
8	4.785577006355862	16	10.93723369430985	24	17.08095953155216
9	5.555545901421528	17	11.70539627317945	25	17.84875825036286

As Table 16 is examined, it is seen that $r_n \cong 2r_{n-1} - r_{n-2}$. As we are searching an approximate value:

$$r_n = 2r_{n-1} - r_{n-2} \tag{4.28}$$

$r_{22} = 2r_{21} - r_{20} = 15.545321768$

Here, if we calculate replacing $2r_{21} - r_{20}$ with $3r_{21} - 2r_{20}$ we will find r_{23} and r_n continues as such.

$r_{22} = 2r_{21} - r_{20} = 15.545321768$

$r_{23} = 3r_{21} - 2r_{20} = 16.313245821$

$r_{24} = 4r_{21} - 3r_{20} = 17.081169874$

$r_{25} = 5r_{21} - 4r_{20} = 17.849093927$

Begin with $n > k$:

$r_n = (n-k+1)r_k - (n-k)r_{k-1}$

Certainly the closest value is $k = n-1$.

$r_{25} = 2r_{24} - r_{23} = 17.848783296$

As k approximates to n, precision increases. (For $k = 10$ $r = 17.869\ldots$) For more precise r_n's:

$$r_n = 3r_{n-1} - 3r_{n-2} + r_{n-3} \tag{4.29}$$

Regarding this:

$r_{23} = 16.313140023$ (16.313135766)

$r_{24} = 17.080963403$ (17.080959531)

$r_{25} = 17.848761767$ (17.848758250)

Using the Pascal's triangle:

$$r_n = 2r_{n-1} - r_{n-2} \tag{4.28}$$

$$r_n = 3r_{n-1} - 3r_{n-2} + r_{n-3} \tag{4.29}$$

$$r_n = 4r_{n-1} - 6r_{n-2} + 4r_{n-3} - r_{n-4} \tag{4.30}$$

$$r_n = 5r_{n-1} - 10r_{n-2} + 10r_{n-3} - 5r_{n-4} + r_{n-5} \tag{4.31}$$

$$r_n = 6r_{n-1} - 15r_{n-2} + 20r_{n-3} - 15r_{n-4} + 6r_{n-5} - r_{n-6} \tag{4.32}$$

$$r_n = 7r_{n-1} - 21r_{n-2} + 35r_{n-3} - 35r_{n-4} + 21r_{n-5} - 7r_{n-6} + r_{n-7} \tag{4.33}$$

$$r_n = 8r_{n-1} - 28r_{n-2} + 56r_{n-3} - 70r_{n-4} + 56r_{n-5} - 28r_{n-6} + 8r_{n-7} - r_{n-8} \tag{4.34}$$

We can continue like same procedure. This recursive sequence $N, N^2, N^3, N^4...$ is natural number sequence. If we calculate r_{25} with respect to each formula

(4.28)	N	17.848783297	$1,2,3,4,5...$
(4.29)	N^2	17.848761767	$1,4,9,16,25...$
(4.30)	N^3	17.848759000	$1,8,27,64,125...$
(4.31)	N^4	17.848758047	$1,16,81,256,625...$
(4.32)	N^5	17.848758319	$1,32,243,1024,3125...$
(4.33)	N^6	17.848758209	$1,64,729,4096,15625...$
(4.34)	N^7	17.848758213	$1,128,2187,16384,78125$
(4.35)	N^8	17.848758095	$1,256,6561,49152,390625...$
(4.36)	N^9	17.848757964	$1,512,19683,196608,1953125...$

$r_{25} = 17.848758250$. If we examine r_n's real values, (4.34) equation provides the best approximation. $r_{25} = 17.848758213$ and $r_{26} = 18.6165350046079$. If we calculate r_{27}, we get that $r_{27} = 19.3842924876935$. (4.35) and (4.36) may give the best result. In future, when r_{26} and r_{27} calculated, we will be able to check. Then we can change the results for bigger r_n's the time real results will show up. When recursive terms are increased, small values are taken into account. Yet for r_n recursive sequences, if n indices getting bigger we can calculate r_n's more precise. We can see this these results via Δr_n. From these results we can approximate via averaging $r_{26}, r_{27}, r_{28}, ...$ values. These values help selection of m values.

$r_{26} = 18.6165350046079 \quad r_{30} = 21.6874761341069$

$r_{27} = 19.3842924876935 \quad r_{31} = 22.4551863093184$

$r_{28} = 20.1520332438645 \quad r_{32} = 23.2228973334240$

$r_{29} = 20.9197600005482 \quad r_{33} = 23.9906192640992$

We may choose the desired r_n's however great as we wish. Making use of the selected value of r_n, we may calculate m in the formula (3.22). For a common solution, if the approximate value of$y$is closer to $4U_n$, we may find y using less operations. Solving the formula (3.21d):

$$m = \frac{2\sqrt{r}}{1+r} \tag{4.21g}$$

In order to give an example of $4U_n$ which is close to y, making use of these formulas:

$10^{25} \Longrightarrow 4u_n = 1666666666666666666666665$

m	r_L	y	
0.448	17.873899438	16664...	$r < r_{26}$
0.430	19.582248647	16501...	$r < r_{28}$
0.400	22.956439237	16255...	$r < r_{32}$
0.375	26.406501178	16070...	$r < r_{37}$
0.250	61.983866769	15386...	$r < r_{83}$

If $m = 0.448$ is selected, y value is getting closer to $4U_n$ but r value that exceeds $r_{25} = 17.848758250$ then $r_{26} > r$. For $m = 0.40$, $r = 22.956439237$ is found. Because of that, we can calculate the count of prime numbers until 10^{32} and $4U_n$ as well as y are still in neighbourhood.

After one year of this work, Professor Doctor Taylan AKDOĞAN on 23 November 2015 conducted a research Two Domains Analysis with a Piecewise Function. For 10^{26}, $r_{26} = 18.616534983044101$...... is given by this research. I calculated before and as mentioned $r_{26} \cong 18.6165350046079$. If it is desired, via replacing real values r_{27}, r_{28}, r_{29} ... can be refined.

4.3 Two Domains Analysis with a Piecewise Function

Prof. Dr. Taylan AKDOĞAN made a table by conducting Two Domains Analysis with a Piecewise Function in order to compare with $\pi(x)$(which is prime counting function). Since we are dealing with 10^{26} level, in stead of local analysis providing (4.26) formula, the table at below is used.

10^n	$\pi(p)$
10^1	4
10^2	25
10^3	168
10^4	1229
10^5	9592
10^6	78498
10^7	664579
10^8	5761455
10^9	50847534
10^{10}	455052511
10^{11}	4118054813
10^{12}	37607912018
10^{13}	346065536839
10^{14}	3204941750802
10^{15}	29844570422669
10^{16}	279238341033925
10^{17}	2623557157654233
10^{18}	24739954287740860
10^{19}	234057667276344607
10^{20}	2220819602560918840
10^{21}	21127269486018731928
10^{22}	201467286689315906290
10^{23}	1925320391606803968923
10^{24}	18435599767349200867866
10^{25}	176846309399143769411680
10^{26}	1699246750872437141327603

x and y values are computed like previous studies. Here, for selecting p value from table is following:

$$
\begin{aligned}
Q &\cong \sqrt{\lambda(p)} \\
n &= \left\lfloor \frac{Q-1}{6} \right\rfloor \\
U_n &= \frac{3n^2+n}{2} \\
\lambda(p) &= \pi(p) - 2 \\
\lambda(q) &= 8U_n - \lambda(p) \\
y &= \frac{\lambda(q)-\lambda(p)}{2\sqrt{1-m^2}} \\
x &= m\sqrt{16U_n^2 - y^2}
\end{aligned}
$$

At these levels, there are two observation 1)$x-y$ quite smooth 2)exponentially increasing. Thus, as model :

$$\log(y_p) = \sum_{n=0}^{n_p} P_n (\log x)^n \tag{4.27}$$

In this study, similarly the previous studies, we choose 8-10 parameter is sufficient and respectively $n_p = 7$ and $n_p = 9$. First study, we examined until 10^{18} level and from 10^{19} to rest as two domains. Second study, we examined until 10^{15} and from rest as two domains. At first domain, we ignored respectively 10^3 and 10^4 in order to avoid fitting problem. m values are selected in order to decrease error margin. Each domain, after fitting, we compute following:

$$
\begin{aligned}
\lambda(p) &= 4U_n - y_p\sqrt{1-m^2} \\
\lambda(q) &= 8U_n - \lambda(p) \\
\Delta\lambda(p) &= \pi(p) - \lambda(p)
\end{aligned}
$$

4.3.1 Results-A

In this case, domain is separated into two parts as $10^5 - 10^{18}$ and $10^{19} - 10^{26}$ in order to find piecewise function. Degree of polynomial is 7.

4.3.1.1 Domain-1: $10^5 - 10^{18}$ and $m = 0.4$

$P_7 = -6.105625159401079 \times 10^{-10}$

$P_6 = 9.950973873330141 \times 10^{-8}$

$P_5 = -6.629678272749457 \times 10^{-6}$

$P_4 = 2.296689313855511 \times 10^{-4}$

$P_3 = -4.275618906402817 \times 10^{-3}$

$P_2 = 3.682667490795615 \times 10^{-2}$

$P_1 = 1.021588443692508$

$P_0 = -8.882512472076487 \times 10^{-1}$

p	$\pi(p) - \lambda(p)$
10^5	0.6821035050470045653388372
10^6	62.960583306942229112814461
10^7	-938.56447281217711691707743
10^8	4080.1683111888793556469731
10^9	21823.387948322294672792735
10^{10}	-74212.517460359963959440682
10^{11}	-1325610.7700098832890439250
10^{12}	-2312308.8597857210164721405
10^{13}	53400648.216125146172784912
10^{14}	320139450.74182554160228920
10^{15}	-1253875620.4807610430352032
10^{16}	-9484100539.8286680834335028
10^{17}	-31519845393.188173728928501
10^{18}	337579736224.10751982822148

4.3.1.2 Domain-2: $10^{19} - 10^{26}$ and $m = 0.35$

$P_7 = 6.350175127325961 \times 10^{-13}$

$P_6 = -2.147435793620114 \times 10^{-10}$

$P_5 = 3.263811245417059 \times 10^{-8}$

$P_4 = -2.873085686399306 \times 10^{-6}$

$P_3 = 1.611132718879993 \times 10^{-4}$

$P_2 = -5.885188624838330 \times 10^{-3}$

$P_1 = 1.153661825964132$

$P_0 = -3.661392127847276 \times 10^{-1}$

10^n	$\pi(p) - \lambda(p)$
10^{19}	-285.339882319770082885409
10^{20}	-5930.226735496480583834941
10^{21}	-5826.531088681402673300124
10^{22}	45539.942134486066509 27923
10^{23}	-163228.4951981004408358521
10^{24}	-5070082.078773085806 17450
10^{25}	-219643141.4121545627628228
10^{26}	2138504726.644095935843745

4.3.2 Results-B

In this case, domain is separated into two parts as $10^5 - 10^{15}$ and $10^{16} - 10^{26}$ in order to find piecewise function. Degree of polynomial is 9.

Domain-1: $10^4 - 10^{15}$ **and** $m = 0.45$

$P_9 = 3.333917444525775 \times 10^{-10}$

$P_8 = -5.929354561974605 \times 10^{-8}$

$P_7 = 4.597389938146314 \times 10^{-6}$

$P_6 = -2.037229809132695 \times 10^{-4}$

$P_5 = 5.678872161482343 \times 10^{-3}$

$P_4 = -1.031522550003682 \times 10^{-1}$

$P_3 = 1.219911741641696$

$P_2 = -9.058325187963698$

$P_1 = 3.947043854733682 \times 10^{1}$

$P_0 = -7.143124212124974 \times 10^{1}$

10^n	$\pi(p) - \lambda(p)$
10^4	2.0063140598561322381230851
10^5	1.1773618587257511735626459
10^6	30.940736997532689833024846
10^7	-532.77930316360427879779897
10^8	5522.4200475918617389709591
10^9	-27507.968360360151200694698
10^{10}	-27089.235103379013005569660
10^{11}	1169119.4192792801580153778
10^{12}	-5954396.9416317249619566217
10^{13}	6515740.7435500984405494704
10^{14}	35234896.564650687712802508
10^{15}	-93247445.362724713142159834

4.3.2.1 Domain-2: $10^{16} - 10^{26}$ and $m = 0.32$

$P_9 = 7.354801431166460 \times 10^{-16}$

$P_8 = -3.116312437405414 \times 10^{-13}$

$P_7 = 5.900531079009955 \times 10^{-11}$

$P_6 = -6.569545596388544 \times 10^{-9}$

$P_5 = 4.760553021525459 \times 10^{-7}$

$P_4 = -2.344945598662795 \times 10^{-5}$

$P_3 = 7.959758615027789 \times 10^{-4}$

$P_2 = -1.845975613300930 \times 10^{-2}$

$P_1 = 1.297269580890030$

$P_0 = -1.014760916282759$

10^n	$\pi(p) - \lambda(p)$	$\lambda(q)/\lambda(p)$
10^{16}	513.86001006706911027573128	10.9372329781445788922707040
10^{17}	-43373.410441280578925450303	11.7053965121778655808037410
10^{18}	1648208.4510577854277226020	12.4734821067023910807635645
10^{19}	-36603967.721622857150797842	13.2415045655889057049829836
10^{20}	523306943.35856390915902192	14.0094736700612398066884165
10^{21}	-4991934357.2776137878278787	14.7773976966152521811555105
10^{22}	31741772172.932343493819157	15.545283313507900325663930
10^{23}	-129464780102.94776735005432	16.3131357557735709720606777
10^{24}	305505654571.74964482936644	17.0809595353321016525404478
10^{25}	-307829801287.02975446521652	17.8487582495219911373419291
10^{26}	-37678838570.541789124125023	18.6165349831332098533263656

Next Step

According to Result B domain 2 the fitting function, $10^n = 10^{27}$ is approximately calculated by extrapolation. The value is following:

$\lambda(p) = 16355556315113781687488005$

Chapter 5

Conclusion

This study starts with the examining U_n sequence that generates pentagonal number. We focused on prime counting function and distribution of primes. Then we discussed the relation between prime numbers and composite numbers via major axis of ellipse. We provided guess of 10^{27} level which is unknown. I hope that curious mathematicians interested in this subject may improve these results.

Chapter 6

Appendix

All lists and table are given in usb flash drive.

1) Table A
2) List A $(m = 0.50)$
3) List B $(m = 0.40)$
4) List C $(m = 0.32)$
5) List D $(m = 0.35)$
6) List E $(m = 0.45)$